Interference Management in Heterogeneous Cellular Networks

by

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Abstract

Heterogeneous cellular networks (HetNets), comprising one or more tiers of small base stations (BSs) overlaid on the tier of well-planned macro BSs, have been commonly recognized as a cost-effective solution to cater to explosive demands on mobile data traffic. However, HetNets also bring some unique challenges such as unplanned deployment of small BSs and increased complexity of interference management. Interference in HetNets becomes one of the major obstacles to capacity improvement and coverage enhancement.

In this thesis, analytical approaches are developed to investigate interference statistics and the performance of 2-tier HetNets including coverage probability and spectral efficiency in mathematical expression, and some insights are revealed on the relationships of interference statistics and network performance with system parameters such as frequency reuse factor, transmission probability, and the signal-to-interference ratio (SIR) gap from Shannon capacity. Specifically, the interference and performance impact of Rician fading and lognormal shadowing are analytically investigated, first based on the Poisson point process (PPP) model where the locations of BSs in each tier are modeled as a PPP. Second, a hybrid model of HetNets is established which considers both location regularity of macro BSs and topological randomness of small BSs, where the coverage probability and the spectral efficiency are derived under different propagation conditions and the significance of macro BS deployment planning is revealed. Third, based on the aforementioned analyses, a well-known interference management technique of fractional frequency reuse (FFR) applied in the tier of macro BSs is studied in detail, and its optimal parameter settings in terms of spectral efficiency and suitable environments are identified.
Finally, a particular interest is attached to multiple antenna technologies including classical beamforming and transmit diversity, which are studied with consideration of the impact of intra-tier and inter-tier interference in HetNets.
Acknowledgements

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To my family
Table of Contents

ABSTRACT .................................................................................................................. II

ACKNOWLEDGEMENTS ............................................................................................... IV

DEDICATION .................................................................................................................. V

TABLE OF CONTENTS ................................................................................................. VI

LIST OF TABLES ........................................................................................................... X

LIST OF FIGURES ....................................................................................................... XI

LIST OF SYMBOLS ....................................................................................................... XIV

LIST OF ABBREVIATIONS ............................................................................................. XVI

CHAPTER 1: INTRODUCTION ....................................................................................... 1

1.1 BACKGROUND AND MOTIVATION .................................................................. 1

1.2 PROBLEM STATEMENT ...................................................................................... 4

1.3 LITERATURE REVIEW ....................................................................................... 6

  1.3.1 Interference management strategies in HetNets ........................................... 7

  1.3.2 Performance analysis of HetNets under Rician fading .............................. 10

  1.3.3 Performance analysis of HetNets under lognormal shadowed Rayleigh fading ................................................................. 13

  1.3.4 Performance analysis of HetNets with hexagonal tessellated macro BSs ... 15

  1.3.5 Fractional frequency reuse in HetNets ..................................................... 19

  1.3.6 Performance analysis of HetNets with multiple antennas ......................... 21

1.4 THESIS OBJECTIVES AND CONTRIBUTIONS ........................................... 23

1.5 THESIS ORGANIZATION .................................................................................... 33

CHAPTER 2: SPATIAL DEPLOYMENT OF BASE STATIONS AND INTERFERENCE MODELS IN CELLULAR NETWORKS ................................................................. 35

2.1 INTRODUCTION ................................................................................................. 35

2.2 SPATIAL DEPLOYMENT OF BASE STATIONS AND FREQUENCY REUSE ................................................................. 36

2.3 INTERFERENCE MODELS IN STOCHASTIC GEOMETRY ................................ 39

2.4 MATHEMATICAL BACKGROUND OF POISSON POINT PROCESS AND HARD-CORE POINT PROCESS ................................................. 42

  2.4.1 Notations ..................................................................................................... 42

  2.4.2 Poisson point process .................................................................................. 43
2.4.3 Hard-core point process ........................................................................................................45
2.5 Summary .....................................................................................................................................45

CHAPTER 3: PERFORMANCE ANALYSIS OF PPP-MODELED HETNETS OVER FADING CHANNELS ........................................................................................................................................47

3.1 Introduction ....................................................................................................................................47
3.2 System Model ..................................................................................................................................48
3.3 Performance Analysis of 2-Tier HetNets with Rician Fading ....................................................54
  3.3.1 Exponential-series approximation to non-central Chi-squared distribution .........................54
  3.3.2 Laplace transform of tier interference with Rician fading ....................................................57
  3.3.3 Coverage probability ............................................................................................................59
  3.3.4 Spectral efficiency ................................................................................................................61
  3.3.5 Numerical results and discussion ........................................................................................63
3.4 Performance Impact of Shadowing in 2-tier HetNets with Rayleigh Fading ............................72
  3.4.1 Laplace transform of tier interference with lognormal shadowed Rayleigh fading .............72
  3.4.2 Coverage probability in lognormal shadowed Rayleigh fading environment .......................73
  3.4.3 Numerical results and discussion ........................................................................................74
3.5 Summary ........................................................................................................................................79

CHAPTER 4: PERFORMANCE ANALYSIS OF HYBRID-MODELED HETNETS WITH HEXAGONAL TESSELLATED MACRO BASE STATIONS OVER FADING CHANNELS ........................................................................81

4.1 Introduction ....................................................................................................................................81
4.2 System Model ..................................................................................................................................82
4.3 Performance Analysis of 2-Tier HetNets over Fading Channels ................................................84
  4.3.1 Approximation of interference statistics from hexagonal tessellated macro BSs ..................84
  4.3.2 Probability of considered user at $r_1$ associating to each tier .............................................87
  4.3.3 Coverage probability of 2-tier hybrid-modeled HetNets ........................................................87
  4.3.4 Coverage probability of homogeneous networks comprising hexagonal tessellated macro BSs .................................................................90
  4.3.5 Spectral efficiency ................................................................................................................92
4.4 Numerical Results and Discussion ...............................................................................................93
  4.4.1 Validating proposed analysis through simulation results ....................................................94
  4.4.2 Coverage probability trend with densifying small BSs .......................................................97
  4.4.3 Spectral efficiency and fairness with densifying small BSs ................................................99
  4.4.4 Performance impact of Rician fading ................................................................................101
  4.4.5 Performance impact of shadowing .....................................................................................103
  4.4.6 Performance impacts of reuse factor and transmission probability ................................104
4.4.7 Comparison of proposed approach with fluid model

4.5 Comparison of Hybrid-modeled HetNets with PPP-modeled HetNets

4.5.1 Comparison of analyses - a unified framework with differences in user distribution and interference characteristics

4.5.2 Performance comparison between PPP-modeled and hybrid-modeled HetNets

4.6 Summary

CHAPTER 5: PERFORMANCE ANALYSIS AND PARTITIONING OPTIMIZATION FOR FRACTIONAL FREQUENCY REUSE IN HETNETS

5.1 Introduction

5.2 System Model

5.3 Performance Analysis of HetNets with FFR

5.3.1 Probability of users in central region when users associate to Macro BSs

5.3.2 Tier interference models of HetNets with FFR

5.3.3 Coverage probability and spectral efficiency

5.4 Numerical Results and Discussion

5.4.1 Validating proposed analysis through simulation

5.4.2 Trade-off between coverage probability and spectral efficiency

5.5 Optimal Partitioning of FFR in terms of Spectral Efficiency for Hybrid-modeled HetNets

5.5.1 Impact of transmission probability on FFR

5.5.2 Impact of shadowing on FFR

5.5.3 Impact of Rician fading on FFR

5.5.4 Impact of SIR gap on FFR

5.5.5 Impact of densifying small BSs on FFR

5.6 Summary

CHAPTER 6: PERFORMANCE ANALYSIS OF HETNETS WITH MULTIPLE ANTENNAS
6.5.1 Validating proposed analysis through simulation .......................................................... 159
6.5.2 Performance comparison between hybrid model and PPP model ..................................... 160
6.5.3 BF performance in hybrid-modeled HetNets .................................................................... 164
6.5.4 Performance of TD and comparison with BF in PPP-modeled HetNets ............................. 166

6.6 SUMMARY .......................................................................................................................... 168

CHAPTER 7: CONCLUSIONS ..................................................................................................... 170

7.1 MAJOR RESEARCH FINDINGS ......................................................................................... 170
7.1.1 Performance analysis of PPP-modeled HetNets with presence of LOS and Shadowing ..... 170
7.1.2 Performance analysis of hybrid-modeled HetNets with hexagonal tessellated macro BSs ....... 171
7.1.3 Performance analysis and partitioning optimization for FFR in HetNets ......................... 172
7.1.4 Performance analysis of HetNets with multiple antennas .............................................. 172

7.2 THESIS CONCLUSIONS .................................................................................................... 173

7.3 ENGINEERING SIGNIFICANCE OF THESIS FINDINGS AND CONCLUSIONS ................... 175

7.4 THESIS LIMITATIONS AND SUGGESTIONS FOR FUTURE WORK ................................. 176
7.4.1 Thesis limitations ........................................................................................................... 176
7.4.2 Suggestions for future work .......................................................................................... 177

REFERENCES .......................................................................................................................... 181

APPENDICES .......................................................................................................................... 196

A1 APPENDICES TO CHAPTER 3 ............................................................................................ 196
A2 APPENDICES TO CHAPTER 4 ............................................................................................ 204
A3 APPENDICES TO CHAPTER 6 ............................................................................................ 208
List of Tables

Table 1.1 Contributions, respective chapter number and associated publications ..................... 25
Table 3.1 Assumed System Parameter Values .............................................................................. 62
Table 3.2 Coefficients of Exponential-series approximation for Rician Fading ....................... 63
Table 3.3 Assumed System Parameter Values .............................................................................. 75
Table 4.1 Assumed System Parameter Values .............................................................................. 93
Table 4.2 Hybrid-modeled HetNets for the typical user at the distance \((r_1, r_2)\) to the nearest
Macro BS and to the nearest small BS ...................................................................................... 109
Table 4.3 PPP-modeled HetNets for the typical user at the distance \((r_1, r_2)\) to the nearest
Macro BS and to the nearest small BS ...................................................................................... 109
Table 5.1 Interference Model in Hybrid-modeled HetNets for the considered user at the
distance \((r_1, r_2)\) to the nearest Macro BS and to the nearest small BS ............................... 128
Table 5.2 Interference Model in PPP-modeled HetNets for the considered user at the distance
\((r_1, r_2)\) to the nearest Macro BS and to the nearest small BS ........................................... 128
Table 5.3 Assumed System Parameter Values .............................................................................. 132
Table 6.1 Assumed System Parameter Values .............................................................................. 159
Table 6.2 Coefficients of exponential-series approximation for \(\gamma(M, 1/M)\) ......................... 160
**List of Figures**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1.1</td>
<td>Evolution directions of LTE-A to future radio access networks</td>
<td>1</td>
</tr>
<tr>
<td>Fig. 1.2</td>
<td>Interference in a HetNet with shared frequency configuration</td>
<td>5</td>
</tr>
<tr>
<td>Fig. 1.3</td>
<td>Interference management strategies in downlink HetNets at the BS end</td>
<td>8</td>
</tr>
<tr>
<td>Fig. 2.1</td>
<td>Hexagonal geometry of BSs with frequency reuse in the regular pattern</td>
<td>37</td>
</tr>
<tr>
<td>Fig. 2.2</td>
<td>Randomly deployed BSs with frequency reuse in the random pattern</td>
<td>38</td>
</tr>
<tr>
<td>Fig. 2.3</td>
<td>Shot noise models categorized by the region containing the interferers</td>
<td>40</td>
</tr>
<tr>
<td>Fig. 3.1</td>
<td>A PPP-modeled HetNet with randomly located macro BSs and small BSs</td>
<td>48</td>
</tr>
<tr>
<td>Fig. 3.2</td>
<td>CCDF of the desired signal power under Rician fading</td>
<td>64</td>
</tr>
<tr>
<td>Fig. 3.3</td>
<td>Impact of Rician fading on coverage probability under Rician-Rayleigh fading</td>
<td>66</td>
</tr>
<tr>
<td>Fig. 3.4</td>
<td>Impact of Rician fading on spectral efficiency and distance distribution from a user to serving BS in each tier</td>
<td>67</td>
</tr>
<tr>
<td>Fig. 3.5</td>
<td>Comparison of coverage probability with Rician-Rayleigh fading and Rician/Rician fading in the tier of small BSs</td>
<td>69</td>
</tr>
<tr>
<td>Fig. 3.6</td>
<td>Frequency reuse impact on coverage probability and spectral efficiency in the small BSs tier of HetNets (Results for Rayleigh fading are shown for comparison with that for Rician fading.)</td>
<td>70</td>
</tr>
<tr>
<td>Fig. 3.7</td>
<td>Impact of frequency reuse on system spectral efficiency in the small BS tier of HetNets</td>
<td>71</td>
</tr>
<tr>
<td>Fig. 3.8</td>
<td>Impact of shadowing on location-dependent coverage probability and spectral efficiency in the macro BS tier of HetNets</td>
<td>76</td>
</tr>
<tr>
<td>Fig. 3.9</td>
<td>Tier coverage probability and spectral efficiency comparison for different frequency reuse factor settings under lognormal shadowed Rayleigh fading</td>
<td>77</td>
</tr>
<tr>
<td>Fig. 3.10</td>
<td>Impact of transmission probability in macro BSs of HetNets under lognormal shadowed Rayleigh fading</td>
<td>78</td>
</tr>
<tr>
<td>Fig. 4.1</td>
<td>A hybrid-modeled HetNet with hexagonal tessellated macro BSs and randomly positioned small BSs</td>
<td>82</td>
</tr>
</tbody>
</table>
Fig. 4.2 Hexagonal geometry of macro BSs with frequency reuse in the regular pattern .......... 83

Fig. 4.3 Approximate the interference from macro BSs as being generated by fictitious BSs
   which are randomly located outside of a disc as a spatial PPP ............................................. 85

Fig. 4.4 Validation of analytical results by simulation results for hybrid-modeled HetNets ...... 96

Fig. 4.6 Spectral efficiency and fairness performance with increasing intensity of small BSs .. 100

Fig. 4.7 Performance impact of Rician fading ................................................................. 102

Fig. 4.8 Performance impact of shadowing ......................................................................... 103

Fig. 4.9 Difference of performance impacts between reuse factor and transmission
   probability for a homogeneous network with hexagonal tessellated macro BSs ........... 105

Fig. 4.10 Spectral efficiencies calculated by the fluid model and the proposed approach in
   Rayleigh fading situation .................................................................................................... 106

Fig. 4.11 Performance comparison of macro BSs between random deployment and hexagonal
   tessellation under Rayleigh fading .................................................................................. 110

Fig. 5.1 Macro BSs with FFR (1,3) for Hybrid-modeled and PPP-modeled HetNets .......... 118

Fig. 5.2 Performance evaluation of the tier of Macro BSs in the Hybrid-modeled HetNet with
   FFR(1,3) .......................................................................................................................... 136

Fig. 5.3 Performance evaluation of the tier of small BSs in the Hybrid-modeled HetNet with
   FFR(1,3) .......................................................................................................................... 138

Fig. 5.4 Performance in the PPP-modeled HetNet with FFR(1,3) ........................................... 139

Fig. 5.5 System spectral efficiency and tier coverage probability in the hybrid-modeled
   HetNets with FFR(1,3) under the Rayleigh fading environment ........................................... 141

Fig. 5.6 Impact of transmission probability on system spectral efficiency improvement of
   FFR(1,3) with the optimal partitioning compared to the universal frequency reuse ........ 142

Fig. 5.7 Impact of shadowing on system spectral efficiency improvement of FFR(1,3) with
   the optimal partitioning compared to the universal frequency reuse ............................. 143

Fig. 5.8 Impact of Rician fading on system spectral efficiency improvement of FFR(1,3) with
   the optimal partitioning compared to the universal frequency reuse ............................. 145

Fig. 5.9 Impact of SIR gap on system spectral efficiency improvement of FFR(1,3) with the
   optimal partitioning compared to the universal frequency reuse ....................................... 146
Fig. 5.10 Impact of increasing intensity of small BSs on system spectral efficiency
improvement of FFR(1,3) with the optimal partitioning compared to the universal
frequency reuse ................................................................................................................................... 147

Fig. 6.1 Performance comparison between the hybrid model and the PPP model with
increasing small BS to macro BS intensity ratio, $\lambda_2/\lambda_1$ .......................................................... 162

Fig. 6.2 Coverage probability and spectral efficiency in the hybrid modeled HetNet with
beamforming, parameterized with increasing number of transmit antennas ......................... 165

Fig. 6.3 Coverage probability and spectral efficiency of transmit diversity in the PPP
modeled HetNet under the independent fading condition, compared to BF under the
fully correlated fading condition ........................................................................................................... 167
List of Symbols

\( P_r(\cdot) \) Probability

\( E_X[\cdot] \) Expectation of random variable \( X \)

\( j \) Index of tier for interference

\( l \) Index of the associated tier for the considered user

\( \phi_j \) Positions of BSs in the tier \( j \)

\( \lambda_j^{BS} \) Intensity of BSs in the tier \( j \)

\( \lambda_{US} \) Intensity of the homogeneous PPP as modeling the locations of users

\( \rho_j \) Frequency reuse factor in the tier \( j \)

\( P_j \) Transmission power of BSs in the tier \( j \)

\( \eta_j \) Transmission probability of BSs in the tier \( j \)

\( B_j \) Association biasing factor of the tier \( j \)

\( r_j \) Distance of the BS closest to the considered user in the tier \( j \)

\( r_{j,i} \) Distance of the BS \( i \) in the tier \( j \)

\( d_j \) Radius of interference free disc in the tier \( j \)

\( R_M \) Apothem of the hexagon if BSs are (equivalently) hexagonal tessellated

\( R_j \) Radius of user located area in the tier \( j \)

\( M \) Number of terms (i.e. for exponential series approximation); or number of antennas

\( N \) Number of terms (i.e. for exponential series approximation)

\( m \) Index of terms / Index of antennas

\( n \) Index of terms

\( X \) Power variation due to shadowing and fading

\( X^{BF} \) Power variation due to fading and BF precoding with multiple antennas

\( X^{TD} \) Power variation due to fading and TD precoding with multiple antennas

\( \bar{h} \) Received power from the associated BS without path loss, no shadowing and fading
\( h \) instantaneous desired received power from the associated BS, including path loss, shadowing and fading \( h = \bar{h} \cdot X \)

\( \bar{g} \) Received power from the interfering BS without path loss, no shadowing and fading

\( g \) instantaneous received interfering power from a BS, including path loss, shadowing and fading \( g = \bar{g} \cdot X \)

\( \mu \) Power variation due to shadowing

\( \sigma_j \) Standard deviation of lognormal (nature base) shadowing in the tier \( j \)

\( \sigma_{j, dB} \) Standard deviation of lognormal (base of 10) shadowing in the tier \( j \)

\( K \) Rician factor

\( K_l \) Rician factor of the desired signal in the tier \( l \)

\( K_{l,j} \) Rician factor of the interfering signal from the tier \( j \)

\( I_0(\cdot) \) Modified Bessel function of the first kind and zero order

\( \bar{P}_j = \frac{P_j}{P_l} \) Transmit power ratio of the interfering \( j^{th} \) tier to the serving \( l^{th} \) tier

\( \bar{B}_j = \frac{B_j}{B_l} \) Association bias ratio of the interfering \( j^{th} \) tier to the serving \( l^{th} \) tier

\( \bar{\alpha}_j = \frac{\alpha_j}{\alpha_l} \) Path loss exponent ratio of the interfering \( j^{th} \) tier to the serving \( l^{th} \) tier

\( I_j \) Aggregate interference from the tier \( j \)

\( G \) SIR gap from Shannon capacity

\( \tau_j \) Spectral efficiency in the tier \( j \)

\( \text{}_2F_1(a,b; c; z) \) Gauss-Hypergeometric function

\( D_1 \) Partitioning distance of FFR in the tier of macro BSs

\( b \) Probability that users associating to macro BSs is staying in the central region

\( W \) Total available spectrum in Hz

\( W^c \) Total spectrum in Hz for users at the central region of FFR

\( W^E \) Total spectrum in Hz for users at the edge region of FFR

\( \rho_1^c \) Frequency reuse factor for macro BSs in the central region

\( \rho_1^E \) Frequency reuse factor for macro BSs in the edge region
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF</td>
<td>Beamforming</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>CCDF</td>
<td>Complementary Cumulative Distribution Function</td>
</tr>
<tr>
<td>CRE</td>
<td>Cell Range Extension</td>
</tr>
<tr>
<td>CoMP</td>
<td>Coordinated Multi-Point</td>
</tr>
<tr>
<td>FFR</td>
<td>Fractional Frequency Reuse</td>
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<tr>
<td>HetNet</td>
<td>Heterogeneous cellular Network</td>
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<tr>
<td>HCPP</td>
<td>Hard-Core Point Process</td>
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<tr>
<td>LTE-A</td>
<td>Long Term Evolution-Advanced</td>
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<tr>
<td>LOS</td>
<td>Line-Of-Sight</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
</tr>
<tr>
<td>MU-MIMO</td>
<td>Multi-User MIMO</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non-Line-Of-Sight</td>
</tr>
<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency-Division Multiple Access</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PGF</td>
<td>Probability Generating Functional</td>
</tr>
<tr>
<td>PPP</td>
<td>Poisson Point Process</td>
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<tr>
<td>RAN</td>
<td>Radio Access Network</td>
</tr>
<tr>
<td>SFBC</td>
<td>Space-Frequency Block Code</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal to Interference plus Noise Ratio</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal to Interference Ratio</td>
</tr>
<tr>
<td>SU-MIMO</td>
<td>Single-User MIMO</td>
</tr>
<tr>
<td>TD</td>
<td>Transmit Diversity</td>
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<tr>
<td>WiMAX</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
</tbody>
</table>
Chapter 1: Introduction

1.1 Background and Motivation

Driven by smartphones and new applications, the demand for mobile data traffic almost doubles each year, which turns out to be 1000 times for the next decade [1] [2]. In order to meet this demand, a set of new techniques is identified and categorized into three dimensions as the evolution directions towards next generation radio access networks [1] [2] [3], shown in Fig. 1.1: (1) Network densification, which means deploying more base stations (BSs) and devices into a given area [4]; (2) Spectrum extension, aggregating more frequency bandwidth together and extending to higher and wider frequency bands [5]; and (3) Further improvement of spectral efficiency by using more antennas [6]. Additionally, traffic offloading to other systems such as wireless local area networks (WLANs) is also expected to continue in the future [1] [7]. Among these three dimensions, network densification has been recognized as the main driver to enhance capacity [4] [8], and it is acknowledged that the most effective and economical architecture to densify networks is the heterogeneous cellular network (HetNet) architecture [3] [4] [9] [10].

Fig. 1.1 Evolution directions of LTE-A to future radio access networks [1] [3]
HetNets include different types of access networks or different types of BSs, where small BSs, such as micro BSs [11], femto BSs [8], and WLAN [12] [13], are introduced into existing cellular networks with well-planned macro BSs. Theoretically, the overall capacity scales with the number of BSs deployed in a unit area, and packing more BSs or devices in a given area would offer more frequency reuse and more capacity [2]. HetNets are well recognised to be economically efficient since capital expenditure and operating expenditure reduce radically by deploying more small BSs instead of macro BSs. HetNets are also motivated by the network deployment principle of bringing networks close to users to offer unprecedented capacity [14] [15]. Introducing more small BSs into existing cellular networks with macro BSs provides a potential performance leap, however, which comes with several technical challenges calling for new solutions, including interference management, mobility management, and difficulties in performance analysis [1] [2] [3]. The more small BSs are packed into a given area, the higher the interference which needs to be mitigated. A robust mobility management mechanism is also required as users move across cell-edges more frequently. The promising performance leap can only be achieved by successfully addressing these challenges. In order to evaluate interference management or mobility management strategies in HetNets, it is imperative to develop tractable and rigorous analytical frameworks for HetNets. But, this is challenging due to the high degree of uncertainty in complicated interference situations, unplanned positions of small BSs (i.e., user-deployed femto BSs), and different statistics of channel characteristics [13] [14] [16]. Actually, even in traditional single-tier cellular networks, performance analysis has always been a challenging problem [14].

In the current fourth generation (4G) wireless network standards such as long term evolution-advanced (LTE-A) [9] and worldwide interoperability for microwave access (WiMAX) [10], the
orthogonal frequency-division multiple access (OFDMA) technique is commonly adopted to avoid intra-cell interference among users in a cell because of the orthogonality among the subcarriers used in the same cell. OFDMA allows the entire spectrum to be reused at much closer distances, leading to high spectral efficiency [17] [18]. This motivates the trend of aggressive frequency reuse, however, resulting in an increase in the interference among tiers and within tiers. At the same time, as more small BSs are packed into a given area with aggressive frequency reuse, not only does the desired signal strength increase but also the interference power from other cells, which significantly limits the system capacity and degrades communication quality. Moreover, strong inter-tier interference between macro BSs and small BSs arises from the high imbalance in path loss and transmission power, and intra-tier interference between unplanned small BSs further complicates the interference scenario. Overall, the interference may significantly undermine the quality of service to users, thus interference is well known as a major impairment that limits HetNet performance and thus demands efficient solutions to be sought [2] [15].

This thesis deals with interference management in the downlink of HetNets since the predominant fraction of the overall data traffic typically occurs in the downlink [17] [19] [20]. In this thesis, performance metrics including the coverage probability and spectral efficiency are used to represent the HetNet performance indicators of network reliability and efficiency, respectively [17] [21] [22]. Coverage probability [23] is defined as the probability that a user is able to achieve some threshold signal-to-interference ratio (SIR) when the user travels in the network service area. It is actually equivalent to the complementary cumulative distribution function (CCDF) of SIR. Spectral efficiency [22] or bandwidth efficiency refers to the information rate that can be transmitted over a
given bandwidth in a communication system, and it is important since it measures how efficiently a limited frequency spectrum is utilized in a HetNet.

1.2 Problem Statement

This thesis deals with interference characterization and performance analysis in the downlink of a 2-tier HetNet where small BSs are overlaid with existing cellular networks with well-planned macro BSs. Each tier is characterized by different values of parameters such as BS intensity, transmit power, association biasing factor, traffic load, frequency reuse factor, and radio propagation environment. Frequency is shared among tiers resulting in high spectral efficiency [18] [20] with two types of interference. The inter-tier interference incurs at the user’s receiver of one tier when interfering signals arise from the transmitters of another tier, for example, the interfering signals from macro BSs for the considered user associating to a small BS as shown in Fig. 1.2. The second type of interference is intra-tier interference which incurs at the user’s receiver of one tier arising from other BSs of the same tier. As shown in Fig. 1.2, the interfering signals from small BSs other than the one the user associates to are categorized into this type.

The main problem addressed is to analyze the performance of the 2-tier HetNet with consideration of the intra-tier and inter-tier interference and to capture the interplay of network performance with some important design parameters, thus providing insights on how to mitigate the performance impact of the interference by allocating resources such as spectral, power, and spatial resources among BSs and tiers. Specifically, the impact of four key factors, including practical propagation environments with the presence of line of sight (LOS), non-line of sight (NLOS), shadowing, network deployment with well-planned macro BSs, frequency reuse strategies, and multiple antenna
techniques, on the interference statistics and thus the performance metrics of the coverage probability and spectral efficiency should be investigated.

Location-dependent performance, that is, the performance for the considered user at a given distance to the closest macro BS, is also desired to be derived, since the interference and the performance can vary radically over the coverage area of a BS [17] due to aggressive frequency reuse. Location-dependent performance analyses are also desired to evaluate location-aware interference management strategies, such as fractional frequency reuse (FFR) [15], a small-BS-location-aware interference management [24], the use of radio environment maps for interference management [25], and a location-dependent self-organizing network mechanism [26]. In addition to system parameters related to network deployment, resource allocation, and propagation situations, other system parameters such as heterogeneous user distribution (i.e. users of small BSs should not be
assumed to be distributed over the whole space), traffic load, and SIR gap from Shannon capacity [17] [27], should be considered in order to make analytical results more consistent with realistic situations. The SIR gap from Shannon capacity characterizes the gap between achievable information rate and the Shannon capacity due to the use of practical modulation and coding schemes [17] [27].

Throughout the thesis, intra-cell interference is assumed to be negligible due to the use of OFDMA, channels are block-faded and frequency-flat (equivalently), and signal transmissions are synchronized over the 2-tier HetNet, all of these assumptions are reasonable in the context of current and future cellular communication standards where OFDMA or similar techniques are exploited [1] [9] [10]. The interference limited case is assumed and the impact of additive white Gaussian noise is ignored which is typical in cellular networks, especially for downlink transmission. The analyses and the revealed insights for the 2-tier HetNet in this thesis can be extended to a general case of HetNet with multiple tiers. This thesis considers the 2-tier HetNet instead of a general case of multiple-tier (>2) HetNet [20], since the investigation of the 2-tier HetNets provides enough details on both intra-tier and inter-tier interference, at the same time, limiting to 2 tiers instead of multiple tiers keep the analyses and the discussions in the thesis clear and concise.

1.3 Literature Review

Though simulation driven techniques have been exploited in industry [21] [28] and academia [29] [30] [31], however, simulation driven techniques may be not only computationally intensive, but also hard to capture the joint interplay of network design parameters and to extract in-depth theoretical insights behind various observations and performance trends. Thus, this thesis focuses
on analytical approaches to evaluate the performance of cellular networks and interference management strategies.

In this section, the various techniques applied to mitigating the impact of interference in HetNets and how to evaluate the HetNet performance analytically are first briefly reviewed to provide an outline of the whole picture. And then followed by more detailed literature reviews related to the problem this thesis studies and the gaps in existing knowledge are highlighted.

1.3.1 Interference management strategies in HetNets

Small BSs can be configured to be either open access or closed access. Open access allows nearby cellular users to use small BSs, whereas closed access restricts the use of the small BS to users explicitly approved, which is possibly the case for user-deployed femto BSs [32] and enterprise-deployed small BSs [33]. With open access and the closest BS association, introducing small BSs into existing networks of macro BSs do not worsen the overall coverage performance or the signal to interference plus noise ratio (SINR) statistics in the downlink [34], provided the macro BSs are assumed to be also randomly and independently deployed as small BSs. This provides optimism to HetNet deployments. However, as mentioned in [32] [35], coordinating inter-tier interference may be difficult and with large delay because of limited capacity of the backhaul, such as a digital subscriber line based links [36]. At the same time, more small BSs are expected in future radio access networks to be packed into a given area [2] [3] and the trend of aggressive frequency reuse [17] [18] both increasing the interference which needs to be mitigated.

Recognizing these challenges, intensive research is being conducted in the literature. This thesis focuses on interference management in the downlink of HetNets at the transmitter side, where the related literature is categorized as shown in Fig. 1.3. Notice that interference cancellation techniques
such as interference rejection combining (IRC) [37] and successive interference cancellation (SIC) [38] also can be used to mitigate the interference impact in HetNets at the user end, but, these are beyond the scope of this thesis.

As shown in Fig. 1.3, there are three types of spectrum configurations among tiers of HetNets for interference management [39] [40] [41]. The first one, frequency partitioning allocates independent fragments of spectrum to each tier [40] [42]. In this way, inter-tier interference is avoided, however, this comes at the cost of low spectral efficiency and low maximum throughput. The second is frequency sharing deployment, in which the entire frequency bandwidth is shared with the tier of macro BSs and the tier of small BSs [39] [43] [44] [45]. Though achieving the best spectral efficiency and maximum peak throughput [17] [18], such a scheme could generate an excessive level of inter-tier interference, causing the penalty on coverage and quality performance. The third one is a hybrid method, where the whole spectrum is divided into multiple parts, some dedicated to the macro BSs and/or small BSs, the others shared between macro BSs and small BSs [41] [46] [47].
This thesis studies frequency sharing configuration due to its potential of higher spectral efficiency and higher maximum throughput compared to the other two approaches [17] [18]. For the frequency-shared deployment between tiers, power allocation may be used to mitigate interference. For example, downlink power control for the femto BSs is treated by Li et al. [43]. A closed femto BS creates a downlink coverage hole at the vicinity of the femto BS for other users, and Nihtila and Haikola [44] propose a scheme that femto BSs adapt the transmit power intelligently to keep the intended coverage area in a residential home. Going forward, they point out that the limited coverage of small BSs is the main reason for limited performance gain in HetNets. This motivates research on association strategy - biasing, or referred to as Cell Range Extension (CRE) in LTE/LTE-A [45], whereby users are pushed from macro BSs to small BSs. Analytical results in [34] suppose the same power settings of femto BSs and analyze the average effects over femto BSs location in terms of macro BSs. Aggressive frequency reuse makes SINR statistics heavily depend on user locations from the closest macro BS, which can potentially lead to location-dependent interference management strategies. Thus in [48] [49], maximal intensity and the optimal power setting for small BSs are derived which depend on the small BSs’ locations from a central macro BS. Another location-dependent strategy is proposed in [39] to partition or share frequency between macro BSs and small BSs based on the small BSs’ locations from a central macro BS. These ideas are verified by simulation where the intra-tier interference of macro BSs is ignored. For the tier of small BSs, its tier-owned resource needs to be further allocated to each small BS when they are densely deployed. Bernardo et al. [50] provide greedy schemes but with high probability of being locked in a local optimal point, and Lee et al. [51] consider an adaptive FFR strategy for mitigating inter-tier interference while keeping spectral efficiency as high as possible.
through a centralized clustering algorithm. However, as mentioned in [8], due to the limited capacity of small BS backhaul, and possibly a large number of small BSs, interference management strategies of small BSs should be distributed, scalable, and self-organizing. Thus distributed solutions [52] have been developed in order to autonomously allocate resources efficiently among small BSs.

In terms of uplink, the performance of back-off procedure over LTE-A random access channel is analyzed in [53] and an enhanced random access channel detection scheme by using multipath diversity is also proposed in [54], however, the performance of random access channel and back-off procedure in a LTE-A HetNet is still desired to be analyzed. Recently, the analytical frameworks for the uplink only focus on homogeneous networks. Coupechoux and Kelif [55] propose an analytical approach to the fluid model for the uplink and give insight to the settings of fractional power control parameters without consideration of shadowing and fading. The transmit power of a mobile in the uplink depends on its location due to the uplink power control, leading to challenges on work extension from the downlink to the uplink. However, Novlan et al. [56] demonstrate that this dependence is weak and can be ignored for random BS deployment. With consideration of Rayleigh fading, the Laplace transform of interference statistics is thus derived and outage and average capacity follow [56], based on the tool of stochastic geometry [17] [57].

1.3.2 Performance analysis of HetNets under Rician fading

The performance of a HetNet is very dependent on its radio propagation environment whose multiplicative characteristics is roughly decomposed into three nearly independent propagation phenomena: small scale fading due to multiple paths of transmission, large-scale shadowing caused by obstructions in-between, and distance-dependent path loss because of propagation loss and obstruction average [58] [59]. Furthermore, the fading has been widely modeled in literature as
Rician or Rayleigh distributed based on whether or not a dominant signal component, that is, LOS, exists, respectively [58] [59]. In this sense, Rayleigh fading can be seen as a special case of Rician fading as the dominant signal becomes weaker and its impact can be ignored.

Network performance analysis has always been a challenging problem even in simple traditional single-tier cellular networks [14], since the deterministic locations of BSs in grid tessellation lead to the analysis difficulty of interference characterization [17]. Indeed, the interference statistic can be calculated assuming deterministic locations of several BSs and then other metrics of network performance follow, with consideration of distance-dependent path loss and fading, or all of three propagation phenomena. For example, limited closest interfering BSs are considered and the interference statistic is calculated as a sum of random variables directly in [60] or with the help of Laplace transform in [61]. However, it does not give a closed-form solution and the components of interference from distant BSs are ignored, it becomes computationally prohibitive with increasing number of BSs considered even for single-tier cellular networks. This challenge has inspired the development of simpler models. Among them, a linear method referred to as “Wyner model” [62] [63] is used in a fairly large amount of literature. By approximating the interference with a ratio of the transmit power of neighboring co-channel BSs, analytical results of capacity reveal considerable insights into the performance of real single-tier cellular networks. However, its accuracy is limited when the number of dominant interferers with similar receive power is not large enough, which is the case for OFDMA-based networks [64]. The approximation also fails to capture the statistical nature of wireless propagation such as fading, shadowing, and dramatic variation of the interference statistical property over the coverage area of a BS when aggressive frequency reuse factor is introduced, making its accuracy even worse. By approximating the average aggregate received
interference generated by the deterministic locations of interferers as being generated by interferers that are randomly located outside of a disc, the fluid model [65] [66], another simpler linear model, provides closed-form analytical results for the spectral efficiency of cellular network with hexagonal tessellated macro BSs. However, the fluid model by itself does not consider shadowing and fading. Topological randomness becomes a part of HetNets nature where small BSs including user-deployed femto BSs have been introduced [15] [67]. Modeling the locations of BSs as a spatial point process, for example, PPP, is thus sensible and has become one of the popular models in the literature recently (see [14] and the references therein). By assuming the locations of BSs of each tier follow a PPP model instead of the traditional grid model, several studies have analyzed the network performance and interference management strategies in HetNets with the assumption of Rayleigh fading for both the desired signal and interference signals, for analytic tractability [14] [20]. Indeed, Rayleigh fading is a valid model for NLOS environments as found for macro BSs in dense urban areas. However, Rician fading appears to be a more appropriate model for small BSs where an LOS or specular component likely exists due to short transmission distance [68]. Although the Rayleigh fading assumption can be relaxed by employing Plancherel-Parseval theorem [69], the resulting integrals are involved and, thus, the stochastic geometry approach loses its tractability in general fading channels [70]. Hence, in a 2-tier HetNet where one tier of small BSs overlays the tier of macro BSs, both Rician fading and Rayleigh fading must be considered for realistic performance analysis, which becomes one of the research goals in the thesis. Rician fading is closely approximated by Nakagami distribution, and correspondingly, its power (non-central chi-square distributed) can be closely approximated as Gamma distribution by using moment matching technique, as shown in [71]. In a Poisson field of interferers, as stated in [72], the composite
Rayleigh fading and lognormal shadowing and the summation are approximated as Gamma distribution and then the performance is derived, however, with the cost of accuracy when the shadowing standard deviation increases [73]. Recently, authors in [74] provide an analytical framework to characterize the impact of Rician fading on device-to-device link under heterogeneous networks, expressing the outage probability by multiple layers of infinite series. For device-to-device link, it is assumed that the interferer is distributed over the whole plane without the lower boundary. In this way, the aggregate interference is symmetric α-stable [75] with an elegant characteristic function and high-order derivative of characteristic function can be derived. However, for femto BSs where the considered user is associated with the closest BS, co-channel BSs are distributed outside of a lower boundary and the aggregate interference is not α-stable anymore, which leads to computational complexity on high-order derivative of characteristic function. At the same time, the outage probability in [74] is expressed by multiple layers of infinite series which leads to the impact of system parameters not being straightforward to be observed, and simple analytical expressions with good accuracy is still highly desirable.

1.3.3 Performance analysis of HetNets under lognormal shadowed Rayleigh fading

As aforementioned about radio propagation environment in Section 1.3.2, in addition to small-scale fading, the large-scale shadowing commonly exists in radio propagation environments due to large obstacles such as hills and buildings in dense urban areas, whose impact on network performance and interference management strategies is also desired to be analyzed in this thesis, in order to make the analytical results more consistent with the practical case. In the previous studies, the composite fading and shadowing distribution is approximated as Gaussian [76], and Gamma approximation [72] [77]. However, the accuracy of those approximations is not always valid, as discussed in [78]
Special attention is drawn to the distribution of lognormal sum because the large-scale shadowing in wireless channels is generally modeled by the lognormal probability density function (PDF) and then the interference is distributed as a lognormal sum. There is no known exact distribution expression of lognormal sum, hence, it is usually approximated as one equivalent lognormal random variable in the literature, e.g., see [69], [72], and the references therein. Nevertheless, calculating the mean and variance of lognormal sum is still computationally-intensive, and the accuracy of the approximation depends on the value of the lognormal variance. Without resort to these approximation approaches, the impact of shadowing in a homogeneous network has been evaluated by the method of “propagation process invariance” in [80] [81], or the method of “stochastic equivalence lemma” in [82], where the impact of shadowing is incorporated into a new PPP model by stochastically mapping the original PPP into a new space, and then results are derived equivalently after averaging over the whole cell in the new space, which gives the same network performance even for different shadowing and fading statistics. However, this conclusion is controversial with the results shown in [69] [83] that the larger variance in the shadowing significantly lowers the performance of cellular networks. Also, the approaches of propagation process invariance and stochastic equivalence lemma cannot be exploited to analyze the performance of HetNets with the biased association policy [20]. The biased association policy assumes each user is associated with the closest BS of the associated tier, equivalent with the maximum received power (biased among tiers) after averaging over the composite shadowing and fading [17] [20], different from that in [82] based on instantaneous SIR and in [80] [81] based on SIR with shadowing (only averaged over Rayleigh fading). This will lead to the proposed model and then the conclusions from this thesis are expected to be different from that in [80] [81] [82].
Compared to these previous works, association with the nearest BS may result in less handover overhead. Moreover, using the approaches in [80] [81] [82], location-dependent performance is hard to track and mapping results in additional difficulty to analyze the hard-core point process (HCPP) [84]. As stated in [15] [72], cellular network performance vary radically over the coverage area of a BS with aggressive frequency reuse, thus location-dependent metrics are also highly desired to be analyzed, such as optimal threshold distance designs for different scheduling schemes in FFR-based LTE networks [85], optimal threshold distance for interference mitigation and hybrid frequency assignment in two-tier networks [86], and scheduling based interference models for LTE networks [61]. Moreover, stochastic mapping methods in [80] [81] [82] are not suitable to apply to HetNets with well-planned macro BSs considered in this thesis, which are neither random nor independent.

1.3.4 Performance analysis of HetNets with hexagonal tessellated macro BSs

Macro BSs in cellular networks are well-planned and tessellated in order to minimize the coverage overlap and system cost while achieving the required coverage probability. Thus, the hexagonal tessellation is widely used for macro BSs [58] [21] [87]. The interference statistics can be calculated directly as a sum of random variables [60] or with the help of the Laplace transform [61], and then metrics of network performance follow. Simpler linear method, such as Wyner model [63], the fluid model [65] [66], and the worst case analysis [88] [89], have derived insightful closed-form analytical result of capacity. However, these approaches have their limitations as aforementioned, moreover, they focus on homogeneous cellular networks and they do not consider the topological randomness of small BSs.

Most of the works on performance analysis and interference management in HetNets are built on one of four kinds of models. A first simpler but less complete model is to focus on a single femto
BS (and its associated user) dropped in a network with tessellated macro BSs [35] or only one central macro BS [90], which is mainly used for the inter-tier interference analysis and is probably only accurate in a fairly sparse femto BS deployment, because it does not take intra-tier interference into account.

The second approach is proposed by Wyner [62] deriving the capacity of homogeneous networks where interference is approximated as the ratio of neighbors’ received signal power. Analytical results of capacity achieved in [62] [63] yield considerable insight into the workings of real systems. Recently, Simeone et al. [36] extend Wyner model to HetNets achieving inner and outer bounds of capacity for users in each tier. However, Wyner model fails to capture the statistical nature of wireless channel, and its accuracy becomes limited in the environment this thesis is considered, as stated in Section 1.3.2.

The third approach is to keep the familiar grid model for macro BSs (including the special case of a single macro BS [46]), and to drop multiple small BSs on top of it, either randomly [48], or in a deterministic fashion [46]. For this deployment assumption, most studies are based on time-consuming Monte Carlo simulations [91], or resort to a simplified model consisting of only one central macro BS and many randomly positioned small BSs [92]. Generally speaking, analytical results of this deployment assumption can be achieved only approximately or asymptotically due to the integral difficulty on sum of interference for grid networks [17]. Seol et al. derive the probability distribution of the interference for downlink analytically in homogeneous cellular networks, taking into account lognormal shadowing and multipath Rayleigh [93] and Rician [94] fading, however, by brutal integrations with consideration of limited number of BSs. Instead, Cheikh et al. [95] provide two approximate analyses of SIR, based on Fenton-Wilkinson based method and Central
limit theorem for causal functions method, respectively. Nevertheless, calculating the mean and variance of lognormal sum is still computationally-intensive, and the accuracy of the approximation depends on the value of the lognormal variance, as mentioned in Section 1.3.3. Recently, some ideas similar to the fluid model [65] [66] are also presented in [63] for the uplink and in [77] for the downlink in HetNets. By limiting users to be distributed in a circular area with the considered BS at the center for the uplink [63] and further limiting interferers to be located outside of the circle with an additional guard region for the downlink [77], the authors give good performance approximation for the performance of hexagonal tessellated macro BSs with Rayleigh fading and full frequency reuse. However, these works are not fully developed and the impact of practical elements such as lognormal shadowing, location-dependent property of network performance, and reuse factor, and traffic load are not considered.

The fourth model, which becomes popular recently and is termed as the PPP-modeled HetNet in this thesis, is to drop both the macro BSs and small BSs randomly and independently, as independent PPPs (see the survey [14] and the references therein). PPP is frequently used to model a variety of networks (see [84] [96] and the references therein) and, it is also feasible to model cellular networks, especially for locations of user-deployed femto BSs and locations of mobile users. Recently, innovative works in [17] [20] inspire intensive studies on HetNet performance analyses where stochastic geometry is adopted and tractable analytical results achieved after assuming the locations of both macro BSs and small BSs are randomly and independently deployed. The appealing aspect is that the downlink SINR distribution and then the outage and average throughput can be found explicitly based on this model, as demonstrated in [17] for homogeneous networks and in [13] [34] for heterogeneous networks. By comparison with the results, Andrews et al. [17] declare the PPP
model gives a lower bound on coverage whereas the grid model is optimistic, and both are about equally accurate. However, in practice, there are two concerns to be addressed on the fourth model: indoor users are in the proximity of femto BSs and are not uniformly distributed on the whole space; macro BSs are well-planned and not completely randomly deployed. With PPP-based layout, it may not be possible to capture the advancements of grid-based networks and the significance of network planning [70]. Though macro BSs are not completely deployed in the optimal way as hexagonal tessellation due to environmental, right of way, traffic demand, and cost constraints, however, for the well-planned macro BSs, the locations are neither random nor independent in practice. Still, the hexagonal grid model is widely accepted as a model for the locations of macro BSs in academia [87] [97] and industry [21] [67]. Thus a natural model for two-tier HetNets is to model the small BS locations as a PPP overlaying a regular hexagonal tessellation of macro BSs, as mentioned in [98] and terms as hybrid-modeled HetNets in this thesis. Hence, performance analysis of this hybrid-modeled HetNets with consideration of the location regularity of macro BSs and topological randomness of small BSs, categorized into the third kind of network deployment, is highly desired, and location-dependent property of interference and performance metrics should be captured, which forms one of the goals in this thesis. After that, the performance difference between HetNets with hexagonal tessellated macro BSs and HetNets with randomly deployed macro BSs is also desired to be investigated, both are claimed as the lower and upper bounds on network performance of a real cellular network deployment, respectively [17] [99].
1.3.5 Fractional frequency reuse in HetNets

Managing interference is a major concern in cellular networks [15]. As mentioned in Section 1.3.1, this thesis focuses on frequency sharing configuration in terms of frequency allocation among tiers, due to its potential of higher spectral efficiency and higher maximum throughput compared to the others. After frequency is allocated into tiers, it is then further allocated to BSs in each tier, and this subsection investigates the related works and interference strategies in this field. The key characteristic of a cellular network is the ability to reuse frequencies to increase both coverage and capacity. In practice, frequencies are well-planned as the way macro BSs are deployed, and frequencies are only reused among macro BSs separated with a sufficient distance as the most common strategy to keep the interference caused by other BSs acceptably low [87] [58]. Conventional frequency reuse treats all users in a BS identically and yields lower spectrum utilization due to fewer available channels in each cell [100] [101]. By dividing the whole frequency band into lots of parallel and orthogonal sub-bands, OFDMA adopted in current cellular networks avoids the interference and provides more flexibility and robustness on interference management. For example, edge users are more prone to be influenced by the interference from neighboring BSs, while users close to BSs are more robust against interference due to low path-loss of the desired signal and high path-loss of the interference from neighbors. Thus, it makes sense to use different frequency reuse factors for users in the cell-center and cell-edge regions: a larger frequency reuse factor for the cell-edge users and a smaller frequency reuse factor for the cell-center users. This location-dependent interference management strategy is known as FFR, or called reuse partition [100]. As commented in [101] [102], FFR can achieve aggressive frequency reuse and balance a trade-off between available frequency resources and interference reduction in OFDMA-based
networks. Notice that FFR can be done semi-statically [31] [101], or implemented in the time domain known as enhanced Inter-Cell Interference Coordination (eICIC) in LTE-A [15].

In the literature, outage and capacity for FFR applied to macro BSs were investigated in [103] with consideration of the interference from neighboring several macro BSs. An adaptive FFR strategy is proposed for small BSs in [51] to mitigate the interference among them with consideration of the interference only from neighboring small BSs inside of a building. Several papers on FFR focused on the optimal parameters of FFR by utilizing advanced techniques such as graph theory [104] and optimization theory [31] [105] [106] to maximize network throughput. Variations of FFR for sectorized macro BSs [101] [107] [108] or irregular layout of macro BSs [109] are also proposed for different application situations. These previous works focused on homogeneous networks and utilized the deterministic model of network deployment, provided simulation results in order to validate the proposed solutions. In order to fill the gap of performance analysis of FFR in homogeneous networks, recently the authors in [110] derived the coverage probability and spectral efficiency in homogeneous networks, by assuming macro BSs randomly deployed and users are partitioning into the cell-center/cell-edge regions based on instant SIR after Rayleigh fading, instead of distance partitioning generally considered [31] [104]. The analytical results for FFR in [110], however, included the diversity benefit of two times frequency allocations, since users with lower instant faded SIR over one sub-band will be allocated to another sub-band where Rayleigh fading is assumed independent and instant SIR was recalculated. For this reason, the questionable results were shown in [110] that the coverage of FFR is even better than the corresponding integer (i.e., non-fractional) frequency reuse. Moreover, assuming macro BSs are randomly deployed is not the
case in practice, and performance analysis for homogeneous networks, especially for well-planned macro BSs, is still desired to be analyzed.

In terms of HetNets, there are a few studies published. Considering that FFR is applied with macro BSs, the authors in [111] commented that small BSs in HetNets should utilize different sub-bands from those used in the closest macro BS to minimize interference to/from macro BSs. Instead of FFR used by macro BSs, recently FFR is proposed in [41] to mitigate the inter-tier interference and analytical results are derived in [47]: some sub-bands are reserved for only macro BSs or small BSs in addition to a common group of sub-bands. This actually is the aforementioned hybrid method of spectrum/time configurations among the tiers of HetNets. Different from the topics in these related studies, this thesis works on performance analysis of HetNets with FFR utilized at macro BSs with FFR optimal partitioning under this situation. For FFR utilized macro BSs, when small BSs are introduced and the network shifts to HetNets, the impact of inter-tier interference on FFR benefit and its optimal setting of parameters is still desired, which is analyzed in this thesis.

1.3.6 Performance analysis of HetNets with multiple antennas

While multiple antenna techniques introduce a spatial dimension into wireless communication, cellular networks actually already use the spatial dimension for a long time, such as frequency reuse. When multiple antenna techniques are put into HetNets, they need to be re-investigated carefully from the system level standpoint, instead of at a single BS. It is well known that, compared with single-user multiple input multiple output (SU-MIMO), multi-user MIMO (MU-MIMO) technique is beneficial for improving average spectral efficiency in a standard scenario consisting of a single cell and multiple users. However, as shown by Chandrasekhar et al. [161], SU-MIMO at each tier provides significantly superior coverage and spatial reuse relative to MU-MIMO in a cellular
network situation. And Liu et al. [113] demonstrate that a hybrid SU/MU-MIMO scheme provides significant gains over SU-MIMO in a system-level evaluation, and argue that cell-edge user spectral efficiency may be reduced if MU-MIMO is used exclusively, due to interference growing with multiple beamforming and reduced transmit power allocated to each user. Park et al. [114] enhance the throughput of each tier by optimizing the trade-off between multiplexing gain and multiuser interference based on inter-tier adaptive selection of optimal number of beams using max-throughput scheduler at the macro BSs. All the above results are validated by Monte Carlo simulations. Until recently, analytical approaches [77] [115] are developed in HetNets by assuming Gamma distribution instead of exponential distribution (the power distribution of a Rayleigh faded signal). Both studies in [77] and [115] argued that when multiple antenna techniques such as zero-forcing are used, the power of received signal for intended user and un-intended users is approximately Gamma distributed, especially in the high SINR region. After that, the aggregate interference distribution is further approximated using the Gamma distribution with second order moment matching in [77] to facilitate simplified calculations, and in [115], ordering results for both coverage probability and per user rate in closed form for any BS distribution for the three considered techniques, using novel tools from stochastic orders.

By manipulating signal transmission in the spatial domain, multiple antenna technologies have been used to mitigate the impact of interference, in addition to the capabilities of bringing multiplexing and diversity gains to boost the coverage probability and throughput [19]. This thesis deals with two complementary techniques with low complexity in the forward link, beamforming (BF) [116] [117] and transmit diversity (TD) [116] [118], for using multiple antennas in downlink HetNets. The performance of BF [119] and TD [120] are analyzed in a system with one or two BSs, instead of
HetNets with inter-tier interference and lots of BSs. Recently, combining with massive antennas techniques, millimeter wave communications have been proposed to be an important part of future radio access networks to provide multi-gigabit communication services [1] [5] [121], where the use of a large number of antennas is a potential solution to exploit higher frequency bands for millimeter wave communications [5] [117] [122]. At high frequency bands, antenna elements can be miniaturized, and very large number of antennas can be co-located, and thus flexible and effective techniques with low complexity, such as BF, has been seen as an essential technique by using its array gain to cope with the huge propagation loss that millimeter wave communications suffer from. [1] [5] [121]. Instead of the assumption of zero-forcing used in [77] [115], performance analysis for HetNets with BF and TD [123] are desired to be investigated, which forms the last goal of this thesis.

1.4 Thesis Objectives and Contributions

This thesis develops the analytical approaches to investigate the interference statistics and the performance metrics of coverage probability and spectral efficiency of 2-tier HetNets, with consideration of four key factors, including the presence of LOS and shadowing, well-planned macro BSs, FFR utilized at macro BSs, and multiple antenna techniques such as BF and TD. The derived analytical results capture the relationships of interference statistics and the HetNet performance with key network design parameters. Some insights are extracted from various observations and performance trends on interference management of 2-tier HetNets under different propagation and traffic load situations. Location-dependent performance, that is, the performance for the considered user at a given distance in terms of the closest macro BSs or the closest small BS, are also derived since they vary radically [17] and are highly desired for evaluating location-aware interference management strategies [15] [24] [25] [26].
More precisely, there are four research goals addressed analytically in the thesis. Most of performance analyses focus on Rayleigh fading for analytical tractability [14] [17] [20], however, the performance impact of the presence of LOS (that is, Rician fading) and shadowing, commonly considered in industry [124], is desired to be analyzed, which forms the first goal of this thesis. Though HetNets with randomly located BSs for all tiers are assumed in most of previous studies for analytical tractability [14] [17] [20], which only represents the lower performance bound for a real cellular network [17] [99], the performance analysis of HetNets with hexagonal tessellated macro BSs should be analyzed in order to capture the significance of network planning for macro BS locations in practice and thus to provide insights for HetNet architects and designers, which produces the second goal of this thesis. After the analyses are developed for these first two goals, this thesis comprehensively investigates the performance benefits of fractional frequency reuse (FFR), one of interference coordination techniques [102], under different deployment and propagation situations. In this way, suitable situations for FFR are identified and the optimal region partitioning for FFR in terms of spectral efficiency are investigated. The last goal of this thesis involves multiple antenna techniques in HetNets which are expected to be more extensively exploited in future radio access networks [5] [6], as shown in Fig. 1.1. By concentrating the transmit power in the intended direction, packing more antennas at BSs has the potential to impressively mitigate the interference impact and to significantly boost the coverage probability and throughput. The analyses developed in this thesis basically employ similar steps, summarized as follows. First, the biased association policy decides the probability of associating to each tier for the considered user at a specified location and the radius of the interference-free disc. Then, based on stochastic geometry, the interference statistics of shot noise with an interference-free disc is characterized by
its Laplace transform with a semi-closed form expression. After that, by fitting the exponential series to the power statistics of the desired signal (if needed), the analytical result of coverage probability is expressed as the summation of the interference statistics Laplace transform. Finally, the spectral efficiency is derived with consideration of SIR gap from Shannon capacity.

Table 1.1 Contributions, respective chapter number and associated publications

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Chapter</th>
<th>Publication</th>
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<tbody>
<tr>
<td>Theoretical Contributions</td>
<td></td>
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<tr>
<td>Performance analysis for HetNets under different propagation conditions</td>
<td>3, 4, 6</td>
<td>[73], [79], [125]</td>
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<tr>
<td>Laplace transform of shot noise interference statistics under different situations</td>
<td>3, 6</td>
<td>[73], [79], [125]</td>
</tr>
<tr>
<td>Approximation of aggregate interference from hexagonal tesselated macro BSs</td>
<td>4</td>
<td>[79], [126], [127]</td>
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<tr>
<td>Performance analysis of HetNets with FFR utilized macro BSs</td>
<td>5</td>
<td>[73], [79]</td>
</tr>
<tr>
<td>Applications of exponential-series approximation</td>
<td>3, 6</td>
<td>[125]</td>
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<tr>
<td>Insights on Interference Management</td>
<td></td>
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<tr>
<td>Performance impact of Rician and shadowing</td>
<td>3, 4</td>
<td>[73], [125]</td>
</tr>
<tr>
<td>Network Planning and frequency reuse planning</td>
<td>4, 5, 6</td>
<td>[79], [126]</td>
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<tr>
<td>Suitable environment for FFR applied macro BSs and its optimal region partitioning in HetNets</td>
<td>5</td>
<td>[79]</td>
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<tr>
<td>Interference Avoidance by Multiple Antennas in HetNets</td>
<td>6</td>
<td></td>
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<tr>
<td>Tier frequency partitioning in HetNets with increasing intensity of small BSs</td>
<td>4, 6</td>
<td>[127]</td>
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Based on the analyses developed and insights extracted, the contributions of this thesis are summarized in Table 1.1, which are categorized into theoretical contributions and insights on interference management, described as follows.

A. *Theoretical contributions*

1. *Performance analyses for HetNets under different propagation situations (Chapters 3, 4, & 6)*

With consideration of the intra-tier and inter-tier interference in HetNets, the mathematical expression of coverage probability and spectral efficiency for PPP-modeled HetNets and hybrid-modeled HetNets under Rician fading (Chapters 3, 4) and lognormal shadowed Rayleigh fading (Chapters 3, 4) are derived. The analyses reveal that, tessellated macro BSs reduce the impact of interference for users associating to macro BSs by the increase of minimal distance of users to interfering transmitters, at the same time, tessellated macro BSs also reduce the impact of interference for users associating to small BS, by limiting the users closer to small BSs. With the introduction of co-located multiple antennas into downlink HetNets under Rayleigh fading, two complementary techniques with low complexity, BF and TD, are investigated in Chapter 6 where analytical approaches are developed and performance impact are formulated. The derived mathematical expression of the performance of HetNets under different situations contributes an understanding of the relationships of interference statistics and network performance with the considered system parameters and propagation situations such as BS intensity, BS transmit power, transmission probability of BSs, frequency reuse factor, SIR gap from Shannon capacity, as so on.

As stated in [15] [72], cellular network performance varies radically over the covered area of a BS with aggressive frequency reuse, locations of users play a key role on achievable performance and investigation on location-dependent metrics is important in order to provide better interference
management and user experience at any location [70]. Thus, besides the performance evaluation over a tier, or over a whole network, this thesis puts more attention on location-dependent interference statistics and performance metrics. In addition to propagation parameters, frequency reuse, power allocation among tiers, transmission probability, and SIR gap from Shannon capacity are considered in the proposed analyses, which provides the potential to comprehensively investigate interference management strategies with consideration of different situations, such as the comprehensive investigation on FFR conducted in Chapter 5.

2. Laplace transform of interference statistics under different propagation situations (Chapters 3 & 6)

Since interference is the main performance-limiting factor in HetNets, it is crucial to characterize the interference statistics [128]. The interference statistics of shot noise with interference-free disc around the considered user (see the discussion on interference models in Section 2.3) is characterized by its Laplace transform derived in this thesis under different propagation environments, including Rician fading (Chapter 3), lognormal shadowed Rayleigh fading (Chapter 3), independent Rayleigh fading among co-located multiple antennas with transmission diversity (Chapter 6), and fully correlated Rayleigh fading among multiple antennas with BF applied (Chapter 6). In this way, the dependence of interference statistics on the considered propagation environments and the other system parameters as aforementioned is explicitly captured by the derived mathematical expression of interference statistics Laplace transform.

3. Approximation of aggregate interference from hexagonal tessellated macro BSs (Chapter 4)

The interference from hexagonal tessellated macro BSs is proposed to approximate as being generated by the shot noise interference model with an interference-free disc around the considered
user, and this approximation is verified to have an acceptable accuracy under propagation situations considered in this thesis. This approximation successfully characterizes the location-dependent property of interference with consideration of frequency reuse. The phenomenon that cell-edge users experience a more severe interference from the neighboring macro BSs than users near the cell-center is also captured in the proposed approach, different from that in PPP-modeled HetNets. Based on this approximation, performance analytical approaches for HetNets with hexagonal tessellated macro BSs are developed which successfully captures the significance of network planning of macro BSs and unified with those for HetNets with randomly deployed macro BSs, analyzed using the tool of stochastic geometry. To some extent, the analytical approach here for hexagonal tessellated macro BSs can be seen as a combination of the analytical study on Hard-core point process in [129] and the fluid model in [130].

4. Performance analysis for HetNets with FFR exploited macro BSs (Chapter 5)

The developed analyses of location-dependent coverage probability and spectral efficiency present more details for interference management in HetNets and, based on these, HetNets with FFR applied macro BSs are comprehensively evaluated considering hexagonal tessellated macro BSs or randomly deployed macro BSs, different propagation situations, and the influence of the aforementioned key practical factors. The proposed analyses reveal that, FFR applied at macro BSs of HetNets significantly enhances the coverage probability of macro BSs in the edge region, in addition to considerable interference reduction for users in the second tier.

Moreover, the optimal parameter setting of FFR regions in terms of spectral efficiency is investigated in HetNets with hexagonal tessellated macro BSs. The achieved results reveal that the FFR benefits depend on network deployment and system parameters such as traffic load, SINR gap
from Shannon capacity, and propagation situations. Interestingly, FFR can improve coverage probability with improved spectral efficiency in HetNets with hexagonal tessellated macro BSs under some situations, which is not the case in PPP-modeled HetNet, where FFR provides the significant reduction in interference for edge users, with a loss in spectral efficiency [131].

5. **Applications of exponential-series approximation (Chapters 3 & 6)**

The finite exponential-series approximation to a continuous PDF has been proposed, and it converges to the exact value with the increase of terms (Chapter 3). It can be used to approximate the desired signal power statistics since the coverage probability can be expressed as summation of Laplace transform of the total interference statistics. Numerical results show the efficiency of exponential-series approximation, only 4 to 8 terms of exponential-series provide acceptable accuracy to approximate the power distribution of Rician fading with factor $K \leq 10$ (Chapter 3), the lognormal shadowed Rayleigh fading (Chapter 3), or Gamma distribution for the shape parameter $M \leq 64$ (Chapter 6). In the thesis, the interior-point algorithm with statistic polynomial complexity [132] is used to fit the exponential-series to a continuous probability density function. Notice that, the PDF of the lognormal shadowed Rayleigh fading is expressed as an exponential-series, as shown in eqn. (A1.7.2), by using Gauss-Hermite quadrature in numerical analysis [66], instead of the interior-point algorithm.

**B. Insights on Interference Management**

1. **Performance impact of Rician fading and shadowing (Chapters 3 & 4)**

The performance evaluation of PPP-modeled HetNets and hybrid-modeled HetNets with the impact of the presence of LOS and shadowing gives network designers some insights on how to deal with antenna installation and user association policy with different propagation situations. The presence
of LOS in the desired signal has a significant enhancement for coverage probability when SIR threshold is less than 5dB, though the resulted improvement of spectral efficiency is marginal. For the interfering signals from small BSs, however, the impact of LOS existence is very marginal on both the coverage probability and spectral efficiency, with the assumptions that the interfering signals from macro BS are Rayleigh faded and the number of small BSs in the coverage area of a macro BS on average no more than 10. The performance impact of the shadowing is converse to that of the presence of LOS. The shadowing deteriorates both the coverage probability and spectral efficiency significantly in HetNets with Rayleigh fading and the biased association policy, especially in the range of the standard deviation of shadowing \( \sigma_{dB} > 4dB \). Notice that this conclusion on shadowing is not the case with different user association policies in [80] [81] [82] that lead to the performance of cellular networks without dependence on the severity of the shadowing but more frequent handoff overhead.

2. **BS deployment planning and frequency reuse planning in HetNets (Chapters 4, 5, & 6)**

Regardless of single antenna (Chapter 4) or multiple antennas (Chapter 6) configuration considered in this thesis, the significance of network planning and frequency reuse planning are revealed mathematically by the performance analysis approaches for HetNets with hexagonal tessellated macro BSs, that is, the hybrid-modeled HetNets. The hybrid modeled HetNets produce lower inter-tier and intra-tier interference compared to the PPP-modeled HetNets comprising randomly deployed macro BSs and randomly allocated frequency reuse. Numerical results demonstrate that, in light shadowing environment with heavy traffic load and when the small BS intensity is similar with or less than the macro BS intensity, the hybrid model provides much better coverage probability and spectral efficiency for both tiers of HetNets. This finding reveals useful insights to HetNet
network architects and designers: network planning of macro BSs and frequency reuse planning for homogeneous network and at the initial deployment phase of HetNets provide significant performance advantage. The results also show that this performance advantages decrease with the increasing intensity of small BSs and becomes marginal when there are, on average, about 100 small BSs located in each hexagon. Notice that all the above insights for interference management still hold for HetNets with multiple antennas.

The higher frequency reuse factor in HetNets will alleviate the impact of intra-tier and inter-tier interference, however, with the cost on the spectral efficiency at the system level. Conversely, among integer frequency reuse, the universal frequency reuse provides the best spectral efficiency in HetNets with the cost on the coverage probability. Looking into more details on location-dependent performance metrics, it is found that the spectral efficiency loss due to high frequency reuse factor is marginal or even better for a user far away to its serving BS, which provides the reasoning for location-dependent frequency reuse interference management strategies, such as FFR investigated in Chapter 6, to satisfy the stringent requirements on communications reliability and efficiency.

3. Suitable situations for FFR applied macro BSs and its optimal region partitioning in HetNets (Chapter 5)

FFR applied in macro BSs of HetNets is evaluated and analyses reveal that FFR significantly enhances the coverage probability of macro BSs in the edge region, in addition to the inter-tier and intra-tier interference mitigation for users in the second tier, when the interfering power from macro BSs is dominant in the network. The key parameters on FFR performance and suitable situations to FFR are identified based on the analytical results developed, which provide guidelines to answer
when and how to apply FFR efficiently in HetNets in practice. When the interference is dominated by the interfering power from macro BSs, it is revealed that FFR is more suitable for situations where the load level is relatively high and the standard deviation of shadowing is low. In contrast to the impact of transmission probability, the analyses reveal that the increase of the practical SIR gap from Shannon capacity leads to FFR providing more improvement on system spectral efficiency compared to the universal frequency reuse. Additionally, the optimal partitioning of FFR regions in terms of spectral efficiency is provided, and the trends of the optimal partitioning of FFR regions depending on transmission probability, shadowing standard deviation, and the intensity of small BSs are investigated in detail in the thesis.

4. Significance of multiple antenna techniques in interference management (Chapter 6)

By concentrating the transmit power in the intended direction, BF indeed significantly boosts the coverage probability and spectral efficiency, and the spectral efficiencies on the whole grow a little better than linearly with the number of antennas in the logarithm scale. Instead, TD reduces the power variation of the faded signal and TD with 4 antennas delivers considerable improvement on the coverage probability at the low SIR threshold compared to the single transmit antenna configuration. However, the further increase of antenna number makes a marginal difference on the coverage probability. Moreover, TD does not offer considerable improvement in terms of spectral efficiency. Analytical results are also compared between BF and TD. These results show that, BF under the fully correlated fading situation provides considerable advantage on spectral efficiency and better coverage probability for the SIR threshold $T \geq -5$ dB, compared to TD under the independent fading situation, while the latter delivers diversity gain enhancing the coverage
probability at the low SIR threshold, and provides better coverage probability at about \( T < -5 \text{dB} \) compared to BF.

5. **Tier frequency partitioning in HetNets with increasing intensity of small BSs (Chapters 4 & 6)**

The 2-tier HetNets are recognized consisting of a tier of macro BSs for coverage and a tier of small BSs for capacity in further radio access networks [1] [133]. If so, dedicated resource should be reserved to macro BSs in HetNets with dense small BSs, in order to ensure the whole service area is covered by macro BSs, since with shared resource, densifying small BSs shrinks the coverage areas of macro BSs to very limited local region. This insight holds regardless of use of single antenna configuration or multiple antennas configuration at the BSs.

1.5 **Thesis Organization**

The thesis is organized into seven chapters. Chapter 2 provides the background required for the analytical framework development, including deployment models, interference models, and a mathematical background on stochastic geometry. The impact of shadowing and LOS on the interference statistics, coverage probability, and spectral efficiency in HetNets are investigated in Chapter 3. Chapter 4 develops the performance analysis of HetNets with hexagonal tessellated macro BSs and the significance of network planning for macro BS locations and frequency reuse is thus captured, providing insights for HetNet architects and designers. After that, Chapter 5 comprehensively investigates the performance benefits of FFR under different deployment and propagation situations. In this way, suitable situations for FFR will be identified and the optimal region partitioning for FFR in terms of spectral efficiency will be demonstrated, depending on key system parameters such as transmission probability, shadowing standard deviation, and the intensity of small BSs. The last topic of this thesis involves multiple antenna techniques in HetNets which is
considered in Chapter 6, where the coverage probability and spectral efficiency of downlink HetNets with BF and TD are derived and compared, and the performance difference between the hybrid modeled HetNet and the PPP-modeled HetNet are investigated. Finally, Chapter 7 concludes the thesis by outlining the major findings, conclusions, engineering significance, and limitations along with suggestions for future work. At the end appendices A1, A2, and A3 provide the proofs of the lemmas and propositions in Chapters 3, 4, and 6, respectively.
Chapter 2: Spatial Deployment of Base Stations and Interference Models in Cellular Networks

2.1 Introduction

Modeling of spatial deployment of base stations (BSs) in the next-generation cellular networks has been attracting considerable interest in recent years [84] [134] [135] [136] [137]. Heterogeneous cellular networks (HetNets) comprise multiple tiers of BSs, which are distinguished by their spatial densities, transmit powers, path loss exponents, and biasing factors [20], and some of the tiers are generally deployed regularly while others are not [134]. Thus, this chapter focuses on each tier separately, which is equivalent with the homogeneous cellular network in terms of modeling spatial deployment of BSs and interference modeling. The HetNet performance is the combined effect of the multiple tiers, which are treated together in the remaining chapters of the thesis.

This chapter provides the research background used throughout this thesis for modeling and analyzing interference statistics and the performance of HetNets. The key advantage of stochastic geometry over the traditional methods is its ability to integrate different aspects of the network randomness into a tractable analytical model, including the randomness in the network topology, the randomness in channel characteristics (e.g. shadowing or large scale fading and small scale fading), behaviors of medium access control strategies, and so on [138]. For this reason, this thesis exploits the tools from stochastic geometry to develop a new analytical framework for modeling and investigating the performance of HetNets, and some basic concepts, properties, and theorems of stochastic geometry are summarized in this chapter. In addition, two interference models, including the shot noise model over the entire plane and the shot noise model with an interference-
free disc, are generally used in the literature related to stochastic geometry and introduced in this chapter with the differences between the two models highlighted.

This chapter is organized as follows: Section 2.2 introduces the spatial deployment models of BSs, including the traditional hexagonal grid topology and the recently popular random model where locations of BSs are described as a PPP. How frequency is reused inside of both deployment models is also introduced. Next, two related interference models in stochastic geometry are described in Section 2.3, and their differences are highlighted. After that, concepts, properties, and theorems of stochastic geometry related to this thesis, including PPP and hard-core point process (HCPP), are introduced in Section 2.4. Finally, Section 2.5 provides the summary to briefly describe which of the concepts in this chapter are used in the subsequent chapters of the thesis.

2.2 Spatial Deployment of Base Stations and Frequency Reuse

As a traditional model, hexagonal tessellated BSs are still widely assumed to date as a semi-realistic approximation in both industry [21] [67] and academia [2] [87] [97] to understand and evaluate the performance of a cellular network, especially for macro BSs which are well-planned and placed as regularly as possible, that is, tessellated. In Fig. 2.1, the topology of macro BSs is shown to be hexagonal tessellated and each of the hexagons contains a BS at the center. Corresponding to this model, frequency reuse with a repeating regular pattern is generally applied, also as shown in Fig 2.1. In this regular pattern, adjacent BSs assigned different frequency groups to avoid severe interference among them form a cluster, and the same frequency group is reused among clusters where BSs in the different clusters are sufficiently far apart and the interference among them is tolerable. The number of frequency groups in a cluster, is called frequency reuse factor (or cluster size, number of BSs in a cluster) denoted by $\rho$. Fig. 2.1 shows the illustration with $\rho = 3$, where the
areas in the same color reside co-channel macro BSs and $R_M$ is the apothem of hexagon. The closest distance among co-channel macro BSs is denoted as $D_0 = 2R_M\sqrt{\rho}$ which is maximized with this regular pattern of frequency reuse [87].

![Hexagonal geometry of BSs with frequency reuse in the regular pattern](image)

Fig. 2.1 Hexagonal geometry of BSs with frequency reuse in the regular pattern

The hexagonal geometry of macro BSs is difficult to analyze to derive closed-form expression for the system performance metrics, such as interference statistics or coverage probability, as a result of the inherent complexity [17] [14]. At the same time, the hexagonal geometry is itself still idealized. The introduction of irregularly deployed small BSs in HetNets, especially user-deployed femto BSs [8], makes topological randomness become an important part of network characteristics. Stochastic geometry entails the mathematical study of random objects defined on a Euclidean space and it has been used to model large-scale ad hoc wireless networks for more than three decades. Recently Andrews, Baccelli, and Ganti [17] derived tractable analytical results of coverage probability and data rate in homogeneous cellular networks by assuming locations of BSs as a PPP, inspiring intensive studies following with the complete topology randomness assumption of BS
locations (see [14] and the references therein). An example is shown in Fig 2.2 where the distances among the BSs are random and BSs with varied cell sizes are possibly arbitrarily close to each other. The stochastic geometry of BSs also allows for interference thinning by frequency reuse, in order to mitigate the interference and enhance the performance. The aforementioned regular pattern is not possible in this random deployment [17]. Instead, frequency is assumed to be randomly allocated and reused [17] [20] and then the frequency reuse impact on interference is captured by a marked PPP [57]. Notice that frequency reuse in this way suffers from the fact that adjacent BSs may be allocated in the same frequency even for large reuse factor. As shown in Fig. 2.2, areas in the same color reside co-channel BSs. For the considered user associating to BS A with the green-shaded coverage area, co-channel BSs could possibly be the immediate neighbors of BS A, as illustrated by BS B in Fig. 2.2.

Fig. 2.2 Randomly deployed BSs with frequency reuse in the random pattern
For 2-tier HetNets considered in this thesis, the stochastic geometry is assumed for the tier of small BSs which is commonly accepted. However, for the tier of macro BSs, both hexagonal and stochastic geometry assumptions will be investigated, since both have potential to reveal insights into real systems. In order to reflect the practical benefits of network planning procedure used in cellular network deployment, some repulsive point processes such as HCPP [57], are used in several studies to impose the minimum acceptable distance constraint between BSs [84] [139]. However, the introduction of the minimum separation distance rule results in the loss of independence among points of HCPP thus there are some challenges to be addressed [14], which is beyond the scope of this thesis.

2.3 Interference Models in Stochastic Geometry

It is crucial to characterize interference statistics and rigorous yet simple interference models are required in order to design interference management schemes in HetNets. However, interference modeling has always been a challenging problem even in a simple traditional single-tier cellular network [14]. Assuming that the interfering BSs follow a regular hexagonal grid leads to either intractable results which require massive Monte Carlo simulation [29] or inaccurate results due to oversimplified assumptions for evolved situations [64]. For hexagonal tessellated macro BSs considered, this thesis proposes an approximate analytical approach to bridge this gap in Chapter 4. Instead, this section focuses on the interference models from a tier of HetNets with randomly located BSs. For the interference from each tier including inter-tier interference and intra-tier interference, there are two interference models categorized by the region containing the interference. The first one is the shot noise model over the entire plane [128] [140], that is, the interferers from the considered tier are uniformly and independently distributed over the entire plane, as shown in Fig.
2.3(a) for $I_j$, the interference from tier $j$. Considering distance-dependent path loss and uncertainty of fading, the power of the aggregate interference of this model is univariate symmetric $\alpha$-stable and characterized by its Laplace transform [141] in a very elegant and appealing form as follows,

$$L_{I_j}(s) = \exp(-\gamma s^{\alpha})$$

(2.1)

Fig. 2.3 Shot noise models categorized by the region containing the interferers

where the dispersion parameter $\gamma$ and the exponent parameter $\alpha$ are derived depending on transmission power, distance-dependent path loss model, and uncertainty of fading, as discussed in detail in [75][142]. As exploited in [74], this model is applicable for the interference from the tier of randomly deployed BSs/device for an uncoordinated transmission, since the receiver location of the uncoordinated transmission, that is, the considered device-to-device link in [74], is assumed to
be completely independent to locations of BSs and locations of other device-to-device links. In this way, the interfering macro BSs or the other transmission devices can be closer to the receiver considered (that is, the receiver of the considered device-to-device link) than the serving transmitter (the transmitter of the considered device-to-device link), as shown in Fig. 2.3(a) for example where $\phi_1$ denotes an interfering transmitter closer to the considered receiver than the serving transmitter.

The second model is the shot noise model with an interference-free disc, or called a guard zone in [128], as shown in Fig. 2.3(b), where the locations of interferers are modeled by a PPP on the plane but excludes the interference-free disc around the considered user. As discussed in [142], the aggregate interference statistics of this model is not $\alpha$-stable anymore. Instead, it approximately follows Middleton Class A distribution [143] and its Laplace transform is much more complicated compared to (2.1) for the first model. The shot noise with an interference-free disc has been applied in situations with contention-based media access control protocols or other local coordination techniques to limit the interference, for example, in wireless ad hoc networks [144], in dense WLAN networks [145], and specially in cellular network [17] [20]. In cellular networks with consideration of the closest association policy in a homogeneous cellular network, the considered user associates to the closest BS at the distance $r_l$ thus all other BSs (that is, interferers) have the distance greater than $d_j = r_l$ [17]. When the biased association policy is considered in a HetNet, the interference of tier $j$ is located outside the circle of the radius $d_j > 0$ determined by the biased association policy in [20]. Association policies are indispensable for cellular networks since networks need to decide which BS serves a user when the user moves across cells or initiates its connection. This thesis focuses on HetNets with consideration of the biased association policy, thus the shot noise model with an interference-free disc is adopted whose parameters, such as the intensity of interferers and
the radius $d_j$ of the interference-free disc will be specified in the following chapters. Though the first interference model is occasionally used for performance analysis of BSs in cellular networks for analytical tractability [34], however, as shown in [146], the significant performance difference has been demonstrated when the interference-free disc is imposed as in the second interference model, compared to the first model.

2.4 Mathematical Background of Poisson Point Process and Hard-Core Point Process

This section presents the definitions, properties, and theorems of stochastic geometry involved in this thesis, including PPP and HCPP, based on the material discussed in [57] [147] but without the proofs, for conciseness. Note that the PPP discussed in this thesis is assumed to be stationary and homogeneous due to its analytical tractability, which is the most important spatial point process for modeling of locations of BSs and users in cellular networks to provide the lower bound performance for real network deployment [17] [99]. HCPP is also briefly introduced to provide the background for the modeling and analysis of the real world HetNets with enhanced accuracy but with cost of analytical complexity and challenges [84].

2.4.1 Notations

The 2-dimensional Euclidian space is denoted by $\mathbb{R}^2$ and the set of positive real number is $\mathbb{R}^+_2$. Bold font letters denotes sets (for example, $\mathbf{S}$) whose elements are written in the normal font such as $s_i, i = 1, \ldots, n$. The null set is $\emptyset$ and $n!$ represents the factorial of the integer $n$. $\mathbf{A}$ denotes a bounded region in $\mathbb{R}^2$ and $|\mathbf{A}|$ is the area of $\mathbf{A}$. The distance between the elements $s_i, s_j \in \mathbf{A}$ is represented by $\|s_i - s_j\|$, where $\|\cdot\|$ represents the Euclidean norm operator. A function expressed as $f: \mathbf{A} \to \mathbb{R}^+_2$ or equivalently $f(\mathbf{A})$, implies mapping from the domain set $\mathbf{A} \subset \mathbb{R}^2$ to a real positive number. Specially, $\phi(\mathbf{A})$ represents a point process $\phi$ on $\mathbf{A}$, or the number of points of $\phi$. 
2.4.2 Poisson point process

**Definition 2.1: Poisson point process (PPP):** A point process \( \phi \) in a bounded region \( A \subset \mathbb{R}^2 \) with intensity \( \lambda > 0 \) is Poisson if the following properties hold:

i) The number of points in \( A \subset \mathbb{R}^2 \) has a Poisson distribution with mean \( \lambda |A| \), i.e.,

\[
P_r(\phi(A) = n) = \exp(-\lambda |A|) \left(\frac{(\lambda |A|)^n}{n!}\right)
\]  

(2.2)

ii) The number of points in disjoint sets are independent, i.e., the set \( \phi(A_1) \) is independent to the set \( \phi(A_2) \) for \( A_1 \subset A, A_2 \subset A, \) and \( A_1 \cap A_2 = \emptyset \).

Since PPP is assumed to be stationary and homogeneous in the bounded region \( A \), the distribution of all instances of \( \phi \) are invariant in \( A \). That is, its intensity \( \lambda \) is constant, denoting the average number of points per unit area as follows,

\[
\lambda = \frac{\mathbb{E}[\Phi(A)]}{|A|} = \frac{\mathbb{E}[\Phi(A_1)]}{|A_1|} = \frac{\mathbb{E}[\Phi(A_2)]}{|A_2|}
\]  

(2.3)

Note that based on the properties ii), for any PPP in \( A \), the points of \( \phi \) in any subset region of \( A \), \( A_1 \), still forms a PPP with intensity \( \lambda \) in \( A_1 \).

**Property 2.1: Distribution of points in a PPP:** Let \( \phi(A) \) be the PPP in \( A \subset \mathbb{R}^2 \) with intensity \( \lambda \), conditioned on a fixed number of points \( \phi(A) \), the points are independently and uniformly distributed in \( A \).

Property 2.1 verifies that a randomly picked-up point in \( \phi(A) \) is uniformly distributed in region \( A \), which is exploited to derive the Laplace transform of the aggregate interference from each tier in the subsequent chapters.

**Property 2.2: PPP thinning:** Let \( \phi_0 \) be a PPP with intensity \( \lambda_0 \), mark each point in \( \phi_0 \) with an identifying variable which is independent with probability \( p \) to be active and probability \( 1 - p \) to
be inactive, then a new PPP $\phi$ with intensity $\lambda = p\lambda_0$ is established that represents the active points of $\phi_0$.

Property 2.2 is used to describe the impact of frequency reuse in a random pattern for a cellular network with a randomly deployed BS, as mentioned in Section 2.2.

**Property 2.3: Distance distribution to the nearest point in PPP:** Let $\phi$ be a PPP in $\mathbb{R}^2$ with intensity $\lambda$. For any point $x \in \mathbb{R}^2$, the complementary cumulative distribution function (CCDF) of the distance $d$ from $x$ to the nearest point in $\phi$ is

$$Pr(d > r) = exp(-\lambda\pi r^2)$$  \hspace{1cm} (2.4)

From Property 2.3, the probability density function (PDF) of the nearest point distance $r$ for a PPP is determined as follows: $f_r(r) = \frac{dF_r(r)}{dr} = 2\pi r exp(-\lambda\pi r^2)$ [17], which is used to determine the distance distribution of users to its serving BS in this thesis.

**Theorem 2.1: Campbell’s theorem - sums over PPP:** Let $\phi$ be a PPP with intensity $\lambda$ and the function $f(x)$ maps the points $x \in \phi$ to a positive real number (i.e., $f: \mathbb{R}^2 \rightarrow \mathbb{R}_+$), then

$$\mathbb{E}[\sum_{x \in \phi} f(x)] = \lambda \int_{\mathbb{R}^2} f(x) dx$$  \hspace{1cm} (2.5)

**Theorem 2.2: Probability generating functional – products over PPP:** Let $\phi$ be a PPP with intensity $\lambda$ and the function $f(x)$ maps the points $x \in \phi$ to a positive real number in the range $[0,1]$ (i.e., $f: \mathbb{R}^2 \rightarrow [0,1]$), then

$$\mathbb{E}[\prod_{x \in \phi} f(x)] = exp(-\lambda \int_{\mathbb{R}^2} [1 - f(x)] dx)$$  \hspace{1cm} (2.6)

Both theorems are exploited to derive the Laplace transform of the interference from each tier of HetNets in the following chapters.
Theorem 2.3: Slivnyak theorem: Let the reduced Palm distribution, denoted by $P_0$, be the distribution of a point process conditioned on the existence of a point at $x \in \mathbb{R}^2$, but not counting it, then the reduced Palm distribution equals to the distribution of a homogeneous PPP.

Theorem 2.3 implies that, if an arbitrary point is added to or subtracted from a PPP, the remainder still forms a PPP. Notice that this arbitrary point must be generated or randomly picked-up from the point process considered.

2.4.3 Hard-core point process

Definition 2.2: Hard-core point process (HCPP): Given a point process $\Omega = \{x_i \in \mathbb{R}^2: i = 1,2, ..., n\}$ with intensity $\lambda$, an HCPP $\phi$ is formed by ensuring that $\|x_i - x_j\| \geq d, \forall x_i, x_j \in \phi, i \neq j$, where $d \geq 0$ is the hard-core parameter.

Imposing the minimum separation distance rule leads to more regular distribution of the points for HCPP compared to PPP. HCPP can be exploited to model the locations of BSs in a well-planned deployment environment such as networks with well-planned macro BSs or to analyze interference mitigation strategies such as Carrier Sense Multiple Access protocol. However, the introduction of the minimum separation distance rule results in the loss of independence among the points of HCPP thus Theorems 2.1 to 2.3, valid for PPP and exploited in the subsequent chapters, no longer apply to HCPP.

2.5 Summary

The research background for modeling and analyzing interference statistics originated from a tier of HetNets with randomly deployed BSs is presented in this chapter to support the analyses and discussions in the following chapters of the thesis. Specifically, the traditional hexagonal grid topology, introduced in Section 2.2, is used in Chapters 4, 5, and 6 for well-planned macro BSs in
the hybrid-modeled HetNet where frequency is reused in the regular pattern, while the random spatial model in Section 2.2 is exploited for all small BSs, and for the tier of macro BSs in the PPP-modeled HetNet, where frequency is reused in a random pattern. Due to association policies associating to cellular networks, the shot noise interference model with an interference-free disc around the considered user, discussed in Section 2.3, is used to analyze the tier interference statistics of HetNets. The PPP concepts, properties, and theorems introduced in Section 2.4 are exploited to derive the Laplace transform of the interference statistics under different propagation situations and antenna configurations in Chapters 3 and 6. The concept of the HCPP is related to hexagonal tessellated macro BSs considered in Chapter 4. Overall, the research background outlined in this chapter provides the basis for the system models and the analyses in the following chapters.
Chapter 3: Performance Analysis of PPP-Modeled HetNets over Fading Channels

3.1 Introduction

In a Poisson point process (PPP)-modeled 2-tier HetNet where one tier of randomly deployed small base stations (BSs) overlays one tier of randomly deployed macro BSs, both Rician fading and Rayleigh fading must be considered for realistic performance analysis, which becomes the focus of the first part of this chapter. A finite exponential-series approximation to the probability density function (PDF) of the power in a Rician faded signal is suggested first which has the potential to converge to the exact value with an increasing number of terms. After that, the Laplace transform of the aggregate interference from a tier under Rician fading is formulated and then an analytical result for the performance of HetNets in both Rician/Rician and Rician/Rayleigh environments is derived.

In addition to small-scale fading, the large-scale shadowing commonly exists in radio propagation environments due to the large obstacles such as hills and buildings in dense urban areas, whose impact on network performance and interference management strategies is investigated in the second part of this chapter. Rayleigh fading for non-line of sight (NLOS) situation is considered for small-scale fading, instead of Rician fading for the line of sight (LOS) situation [28], since the existence of LOS leads to the small standard deviation of shadowing [67] [97], resulting in the performance impact of shadowing not being significant.

The remainder of this chapter is organized as follows. The system model is presented in Section 3.2. Analytical results of the coverage probability and spectral efficiency are derived for PPP-modeled HetNets under two propagation situations, Rician fading in Section 3.3 and lognormal shadowed Rayleigh fading in Section 3.4, respectively. Finally, Section 3.5 summarizes this chapter.
3.2 System Model

A downlink PPP-modeled HetNet with 2 tiers of BSs in the unlimited Euclidean plane, as shown in Fig. 3.1 for example, is considered in this chapter, characterized by the following assumptions:

(A1) **BS deployment**: the HetNet consists of the first tier of macro BSs \( j = 1 \) and the second tier of small BSs \( j = 2 \), and the positions of BSs in each tier \( j \ (j = 1, 2) \) are generated by a homogeneous PPP \( \phi_j \) with intensity \( \lambda_j^{BS} \), which implies that the positions of BSs are independent among each other, and the number of BSs in a certain area \( A \) is Poisson distributed with the parameter \( \lambda_j^{BS} A \). Both tiers are further distinguished from each other by radio propagation parameters, association biasing factor, and other parameters, in the following assumptions.

(A2) **Spectrum allocation**: The total available spectrum of \( W \) Hz is shared among the \( L = 2 \) tiers, and the resource in both tiers is open to all the users. Frequency reuse factor \( \rho_j \ (\geq 1) \) determines the number of different frequency groups used in the tier \( j \), and only one frequency group is assigned...
to each BS. Different from a regular pattern of frequency reuse in hexagonal tessellated cellular networks [97], these frequency groups are assumed to be randomly allocated in the PPP-modeled HetNets, since a regular pattern is not possible in a random deployment [14] [17]. Under the assumption of random and independent allocation of frequency groups to each tier, the impact of frequency reuse on the PPP-modeled HetNet performance is depicted completely by PPP thinning, so that the area intensity of co-channel BSs from the tier \( j \) reduces to be \( \lambda_j^{BS}/\rho_j \). An example of the random pattern of frequency reuse is shown in Fig. 3.1 for macro BSs, where areas in the same color reside co-channel macro BSs, and the BS serving the considered user has the common border with its co-channel BS in the blue.

\( A3) \) Power and transmission probability: Each BS in the tier \( j \) transmits with a fixed power of \( P_j \) with the transmission probability \( \eta_j \), where \( \eta_j \) is the probability of a BS in the tier \( j \) transmitting over an assigned channel at any time instant, and it is identical and independent among BSs [57] [148]. \( \eta_j \) relates to traffic load in the tier \( j \) and plays a key role in determining the interference. The maximum transmission probability \( \eta = 1 \) means all the BSs are transmitting concurrently all the time, as considered in [17].

\( A4) \) Path loss, shadowing, and fading: It is assumed that BSs have one transmit antenna each and user devices are also equipped with one receive antenna each. The transmitted signals in the tier \( j \) experience log-distance path loss with the exponent \( \alpha_j > 2 \) [67] [97], and the received average power (averaged over the small-scale fading) from a BS at the distance \( r \) is further shadowed and faded. The shadowing random variable with unit mean is assumed to be lognormal, \( \mu \sim \text{lognormal}(\frac{\sigma_j^2}{2}, \sigma_j) \), representing the random deviation from the log-distance path loss model [97]
is the shadowing standard deviation in the natural base, and \( \sigma_j = \frac{\sigma_{j,\text{dB}}}{10} \log_e(10) \) where \( \sigma_{j,\text{dB}} \) is the standard deviation in \( dB \). The small scale fading power gain is denoted by \( X \) with the mean value \( E(X) = 1 \), and \( E(\cdot) \) is the expectation operator. When the received signal is Rician faded then \( X \) has a scaled non-central chi-squared distribution with two degrees of freedom, whose pdf is \([68]\)

\[
f_X(x) = (1 + K)e^{-K - (1 + K)x}I_0(2\sqrt{K(1 + K)x})
\] (3.1)

where the Rician factor \( K \) is the ratio of the power in the dominant component to the average power in the diffuse components and \( I_0(\cdot) \) is the modified Bessel function of the first kind and zero order. Values of the Rician factor \( K \) in a mobile environment (outdoor and indoor) usually range from 1 to 10 \([67]\). When the received signal is Rayleigh faded, \( X \) is equivalent to Rician faded with \( K = 0 \), leading to \( X \) being exponential distributed, and with \( E(X) = 1 \),

\[
f_X(x) = e^{-x}
\] (3.2)

Notice that the first part of this chapter focuses on the performance impact of Rician fading and no shadowing (setting the shadowing standard deviation \( \sigma_j = \sigma = 0 \)), and the impact of shadowing with Rayleigh fading (setting \( K = 0 \)) is investigated in the second part.

\textbf{(A5) User distribution:} users are distributed uniformly over the whole service area and their positions are modeled as a homogeneous PPP \( \phi_U \) with intensity \( \lambda^U \). In the thesis the user intensity is assumed to be large enough to ensure that each BS has at least one user associated. Further discussion for small \( \lambda^U \) is shown later in Section 3.3.2. Without loss of generality, the considered user is randomly picked up and the coordinate is established with the user at the origin \([17]\) \([57]\).

\textbf{(A6) Biased association policy:} The distance of the BS \( i \) in the tier \( j \) to the considered user is denoted as \( r_{j,i} \) and \( r_j = \min_i(r_{j,i}) \) is called the distance of the user to the closest BS in tier \( j \). The considered
user associates to the tier \( l \) from which it received the maximum biased average power, mathematically

\[
l = \arg \max_{j \in [1, L]} P_j r_j^{-\alpha_j} B_j
\]

(3.3)

where \( B_j \) (\( B_j \geq 1 \)) denotes association biasing factor of the tier \( j \) which enables flexible tier association [20]. The biased association policy offloads data traffic from heavily-loaded macro BSs to lightly-loaded small BSs, but with the cost of increased interference for offloaded users. For the considered user at \((r_1, r_2)\) to the nearest macro BS and the nearest small BS, respectively, as shown in Fig. 3.1, the biased average received power \( P_1 r_1^{-\alpha_1} B_1 \) from the tier \( j = 1 \) is greater than that from \( j = 2 \), when \( r_2 > \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha_2}} r_1^{\frac{\alpha_1}{\alpha_2}} \) which means there is no small BSs inside the circle of the radius \( R_2 = \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha_2}} r_1^{\frac{\alpha_1}{\alpha_2}} \). In this case, the considered user associates to the tier of macro BSs \((l = 1)\) based on the biased association policy. If there was a small BS inside of this circle, the considered user would have associated to the tier of small BSs.

(A7) SIR and spectral efficiency: For analytic simplicity, interference limited situation is assumed and the thermal noise is negligible. Then \( SIR = h / \sum_{j=1}^L I_j \) where \( h = P_j r_j^{-\alpha_j} \mu X \) is the instantaneous received desired signal power from the serving BS in tier \( l \) and \( I_j = \sum_{i \in \Phi_j} g_{ji} \) is the aggregate interference power from tier \( j \). The received power from the interfering BS \( i \) in the tier \( j \) at the distance \( r_{ji} \) is denoted by \( g_{ji} = P_j r_{ji}^{-\alpha_j} \mu_{ji} X_{ji} \) where \( \mu_{ji} \) represent the power variation due to shadowing and \( X_{ji} \) represent the power variation due to fading. No intra-cell interference is considered which is the case for current cellular communications standards such as long term
evolution-advanced (LTE-A) and beyond where orthogonal frequency division multiple access (OFDMA) is popularly adopted.

Adaptive modulation is assumed and the achievable (link) spectral efficiency is approximately modeled by a modified Shannon’s formula

$$\tau = c/W = \log_e(1 + SIR/G)$$ (3.4)

in nats/s/Hz, where $c$ denotes the achievable information rate, $G > 1$ is referred to as the SIR gap from Shannon capacity, which characterizes the gap between achievable information rate and the Shannon capacity due to the use of practical modulation and coding schemes [17] [27]. Note that the use of the modified Shannon’s formula make the analyses developed in this thesis independent of system specific details, such as the particular modulation and coding techniques being used. $\tau' = \tau \times \log_2(e)$ in the unit of bits/s/Hz. The spectral efficiency at the system level, called system spectral efficiency, is $\tau_s = \tau \times \eta/\rho$ since each link only utilizes part of frequency-time resource due to the limitation of frequency partitioning $\rho$ and transmission probability $\eta$.

Validation of system assumptions: While the PPP assumption in (A1) for the small BS locations is reasonable and reflects the real-life conditions [15], the PPP model is atypical for the macro BS locations which are not independent from each other. However, the results on the PPP model serve as a lower bound compared against the idealized hexagonal tessellation which serves as an upper bound, the analyses for both provide theoretical insights into the network performance [17] [20], thus this chapter uses the PPP model for the macro BS locations, and leaves the investigation on the performance impact of hexagonal tessellated macro BSs to the next chapter. Correspondingly to the assumption of completely randomly located BSs in each tier, the randomly distributed spectrum allocations described by (A2) is a reasonable assumption used in the recent literature such as [17]
[20], and is feasible to implement within the existing OFDMA schemes such as in LTE-A networks [67] due to the availability of plenty of frequency-time resource blocks. The regular pattern of frequency reuse [97] will be investigated in the next chapter together with the hexagonal tessellated macro BSs, however, it is not possibly feasible to implement in a random deployment [14] [17]. The transmission power of BSs is mainly fixed for the downlink in current cellular standards such as LTE-A [149], which forms the basis for the assumption of fixed level of power in each tier in (A3). The open access of radio resource among users and tiers is implied in (A5) and (A6) since it provides the best efficiency [17] [20], and the analyses developed in this thesis pave the way to consider more complicated situations with differentiated resource types and users. For example, the 2-tier PPP-modeled HetNets with dedicated radio resource in each tier and additional shared radio resource among tiers can be analyzed as the combination of HetNets, in each HetNet only one type of radio resource is considered. With the assumption of open access of radio resource, the user distribution in (A5) and biased association policy in (A6) are commonly assumed in the literature [20] [20] and the biased association policy become realistic in some cellular network standards such as LTE-A [67].

A general case of radio propagation characteristics is depicted by (A4) which will give the four situations considered in this chapter by the different setting of parameters, Rayleigh fading (all the signals are Rayleigh faded by setting Rician factor $K = 0$, without shadowing by setting $\sigma = 0$), Rician/Rician fading (both the desired and interfering signals are Rician faded, without shadowing by setting $\sigma = 0$), Rician/Rayleigh fading (the desired signal is Rician faded but the interference is Rayleigh faded, both without shadowing by setting $\sigma = 0$), and lognormal shadowed Rayleigh faded (setting Rician factor $K = 0$ and $\sigma \neq 0$). The Rayleigh fading reflects the presence of
obstructions between the BS and the user (i.e. line of sight does not exist) such as dense urban areas and it gives elegant analyses [14] [17] [20]; while Rician/Rician fading characterizes the existence of line of sight for the desired and interfering signals in dense deployments with short distance such as small BSs or indoor deployment [71]. Differentiating the fading characteristics of the desired signal (with a shorter distance) and the interfering signals (with a longer distance) provides reasons to consider Rician/Rayleigh fading [68] [74]. Instead of Rayleigh or Rician fading characterizing the small scale fading, the large scale shadowing also commonly exists in practice due to large obstacles, whose impact is investigated as the last propagation situation together with Rayleigh fading [66] [99].

For notational brevity, the following parameters are defined

\[
\hat{P}_j = \frac{p_j}{p_l}, \quad \hat{B}_j = \frac{b_j}{b_l}, \quad \hat{\alpha}_j = \frac{\alpha_j}{\alpha_l}, \quad \hat{\lambda}_j = \frac{\lambda_j}{\lambda_l}
\]

which are the transmit power ratio, association bias ratio, path loss exponent ratio of the interfering \( j \)th tier to the serving \( l \)th tier, and BS intensity ratio, respectively [20].

### 3.3 Performance Analysis of 2-Tier HetNets with Rician Fading

In this section first the power of the desired Rician faded signal is expressed as an exponential-series. In this way, the coverage probability can be expressed with Laplace transform of interference statistics. After that, the Laplace transform of the aggregate interference from a tier \( j \) is formulated with consideration of Rician fading. Combining these two results developed together, the coverage probability and thus the spectral efficiency are derived for HetNets under Rician fading in Sections 3.3.3 and 3.3.4, respectively, followed by numerical result and discussions in Section 3.3.5.

#### 3.3.1 Exponential-series approximation to non-central Chi-squared distribution

##### A. Exponential-Series Approximation
A non-oscillatory function may be approximated using an exponential-series [150]. And
asymptotically, the modified Bessel function of order \( \nu \) behaves as
\[
I_\nu(x) \sim \frac{e^x}{\sqrt{2\pi x}}, \quad x \in [0, V]
\]  
(3.5)

Thus Rothwell in [150] approximates the product \( e^{-x}I_\nu(x) \) by a finite exponential series within a
finite interval. Note that (3.1) is of the form \( e^{-y}I_\nu(2\sqrt{Ky}) \) which decreases even faster with an
increase in \( y \), where \( y = (1 + K)x \). Now if the form \( e^{-y}I_\nu(2\sqrt{Ky}) \) in (3.1) is replaced by a finite
exponential series, then an interesting approximation \( g(x) \) to a non-central chi-squared distribution
\( f_X(x) \) is defined as follows:
\[
f_X(x) \approx g(x) = \sum_{n=1}^{N} w_n u_n e^{-u_n x}, \quad x \in [0, V]
\]  
(3.6)

where \( N \) is the number of terms in the exponential series and both \( w_n \) and \( u_n \) are the real
coefficients of the \( n \)th term, \( n = 1, 2, ..., N \), and \([0, V]\) is the range of interest for \( x \). In (3.6), \( V \) is a
positive real number and the value of \( N \) is selected to achieve a desired accuracy. Based on (3.6),
the complementary cumulative distribution function (CCDF) of the received desired signal power
\( F_x(x) = \int_{x}^{\infty} f_X(y)dy \) is approximated by
\[
\tilde{G}(x) = \int_{x}^{\infty} g(y)dy = \sum_{n=1}^{N} w_n e^{-u_n x}, \quad x \in [0, V]
\]  
(3.7)

Clearly \( \tilde{F}_x(+\infty) = 0 \) and \( \tilde{F}_x(0_+) = 1 \), which also hold true for its approximation, \( \tilde{G}(+\infty) = 0 \) and
\( \tilde{G}(0_+) = 1 \). The two limits yield constraints on \( w_n \) and \( u_n \), given by:
\[
u_n > 0, \quad n = 1, 2, ..., N
\]  
(3.8)
\[
\sum_{n=1}^{N} w_n = 1
\]  
(3.9)

**Proposition 3.1:** The PDF of \( X, f_X(x) \) in (3.1) can be approximated by the exponential series \( g(x) \)
defined by (3.6) where \( x \) is a value on any interval \([0, V]\), i.e. \( x \in [0, V] \).
**Proof:** Provided in Appendix A1.1.

The proof applies to the case where \( u_n \) takes on a positive real value because \( n = 1, 2, \ldots \) are also positive real numbers, and \( w_n \) is a real number. For this case there exists at least one solution (\( u_n = n \) and \( w_n = w'_n/u_n \)) to make \( |f(x) - g(x)| < \varepsilon \). If all the weights \( w_n \) are further constrained to be positive, the exponential-series approximation becomes a mixture distribution [151] but Proposition 3.1 is no longer valid. When all the weights \( w_n \) are positive, the Hessian of \( g(x) \) is positive definite and thus the approximation (3.6) is strictly convex, which implies that (3.6) cannot correctly approximate some continuous functions such as (3.1), a non-convex function. As such, this thesis assumes that the weights \( w_n \) are any real number.

**B. Determining the coefficients of the exponential-series approximation**

An optimization problem is formulated to determine the coefficients \( w_n \) and \( u_n \) of the exponential-series approximation, comprised of a non-linear objective function defined by:

\[
\min_{w_n, u_n} \int_0^V \left( \tilde{g}(x) - \tilde{f}(x) \right)^2 dx
\]  

(3.10)

with the constraints specified by (3.8) and (3.9). The value of the integration upper limit \( V \) in (3.10) is determined such that \( Pr\{X \geq V\} \leq 10^{-4} \) (i.e. 99.9% likelihood that \( x \in [0, V] \)). The constrained non-linear optimization problem can be solved by standard non-linear optimization methods, such as the interior-point algorithm [132], with statistical polynomial-time complexity. The initial values of \( w_n \) and \( u_n \) are generated as the standard normal distribution, and the solver runs from different sets of initial values to find a good local solution. Since the optimization problem is non-convex, there is no guarantee the local solution will converge to the global optimal. But, as shown in Section
3.3.5, these achievable local solutions still provide a good accuracy of the proposed approximation with \(N = 4\). Some values of the coefficients \(w_n\) and \(u_n\) are provided in section 3.3.5.

### 3.3.2 Laplace transform of tier interference with Rician fading

With joint consideration of the frequency reuse factor \(\rho_j\) and transmission probability \(\eta_j\), the effective intensity \(\lambda_j\) of interfering BSs in the tier \(j\) at any time instant is:

\[
\lambda_j = \frac{\lambda_j^{BS} \eta_j}{\rho_j}
\]  
(3.11)

Eqn. (3.11) can be extended to consider the probability of a BS without any user associated, instead of the previously stated assumption that \(\lambda^U\) is big enough to ensure each BS has at least one user. The average cell area in the tier \(j\) of the HetNet is \(A_j = \Pr\{l = j\} / \lambda_j\) where \(\Pr\{l = j\}\), the probability that users associate to the tier \(j\), has been derived in [20] based on the biased association policy in (3.3). Then each BS in the tier \(j\) has a probability \(p_{j,0}\) without any user associated, where

\[
p_{j,0} = e^{-\lambda^U A_j}
\]  
(3.12)

Thus the intensity of the effective interfering BSs with consideration of reuse factor \(\rho_j\), the transmission probability \(\eta_j\), and the probability of BS without any user associated, \(p_{j,0}\), becomes

\[
\lambda_j = \eta_j (1 - p_{j,0}) \lambda_j^{BS} / \rho_j
\]  
(3.13)

The impact of the probability of BSs without any user associated can be seen as the PPP thinning with a low intensity in (3.13), since BSs without users shall not transmit any signal. In this chapter, \(\lambda^U\) is assumed to be big enough and (3.11) is considered in the following analyses, though these analyses are applicable with consideration of the probability \(p_{j,0}\) as shown in (3.13). The probability \(p_{j,0}\) in (3.12) approaches to 0 and (3.13) converges to (3.11) with the increase of \(\lambda^U\).
Lemma 3.1: When the positions of the interfering BSs (in the tier $j$) are modeled by a PPP $\phi_j$ with the intensity of $\lambda_j$ in an unlimited Euclidean plane excluding the area $b(0, d_j)$, the interference free disc of radius $d_j$ around the considered user, and all the interfering signals are Rician faded with Rician factor $K_I$, the Laplace transform of the aggregate interference $I_j$ received by the considered user is

$$
L_{I_j}(s) = \exp \left\{-2\pi \lambda_j \frac{2}{\alpha_j} \left[ \sum_{n=0}^{\infty} (-1)^{n+1} n! \frac{K_I^n}{d_j^{\alpha_j+n}} B\left(\frac{\nu}{d_j^{\alpha_j+n}}, n+I_{n=0} - \frac{2}{\alpha_j}, \frac{2}{\alpha_j} + I_{n=0}\right) \right] \right\}
$$

(3.14)

where $\nu = \frac{sP_j}{1+K_I}$, the incomplete Beta function is denoted as $B_x(a, b) = \int_0^x z^{a-1} (1 - z)^{b-1} dz$ for $0 \leq x \leq 1$, and the zero/non-zero indicator functions of $n, I_{n=0} = 1$ and $I_{n=0} = 0$ if $n = 0$, otherwise when $n \geq 1$, then $I_{n=0} = 0$ and $I_{n=0} = 1$.

Proof: Provided in Appendix A1.2.

By using the hypergeometric representations of the incomplete Beta function, shown in (8.17.9) in [152], $L_{I_j}(s)$ can be also expressed in the form of Gauss-Hypergeometric function as

$$
L_{I_j}(s) = e^{-2\pi \lambda_j \frac{sP_j}{1+K_I}} \left[ \sum_{n=1}^{\infty} \frac{(-K_I)^n}{n!} \binom{n+\frac{2}{\alpha_j}}{\frac{1}{\alpha_j+n}} \binom{\frac{1}{\alpha_j+n}}{n} 
\right] \left[ 2F_1\left(\frac{\nu}{d_j^{\alpha_j+n}}, n+I_{n=0} - \frac{2}{\alpha_j}, \frac{2}{\alpha_j} + I_{n=0}\right) \right]
$$

(3.15)

Setting $K_I = 0$, Lemma 3.1 gives $L_{I_j}(s)$ with Rayleigh fading as follows, which is consistent with the previous result in [20].

$$
L_{I_j}(s) = \exp \left\{-\frac{2\pi \lambda_j sP_j}{\alpha_j} \frac{d_j^{\alpha_j-2}}{\alpha_j-2} \right\} 2F_1\left(1, 1 - \frac{2}{\alpha_j}; \frac{2}{\alpha_j}; -sP_j d_j^{-\alpha_j}\right)
$$

(3.16)
3.3.3 Coverage probability

**Proposition 3.2:** When the desired signal from the serving BS in the tier $l$ experiences Rician fading with factor $K_l$, and the interfering signals from the tier $j$ experience Rician fading with factor $K_{l,j}$, the coverage probability of a considered user at the distance $r_l$ from its serving BS is:

$$p_{c,l}(T|r_l) = \sum_{n=1}^{N} w_n e^{-\pi \sum_{j=1}^{2} \lambda_j \tilde{P_j}} Z(u_n T, \alpha_j, \tilde{\eta}) r_l^{2/\alpha_j}$$

(3.17)

where

$$Z(u_n T, \alpha_j, \tilde{\eta}) = 2 \left( \frac{u_n T}{1+K_{l,j}} \right)^{2/\alpha_j} \sum_{n=0}^{\infty} \frac{(-1)^n + \eta_{n \neq 0}}{\alpha_j^n n!} B_{x_j} \left( n + I_n = 0 - \frac{2}{\alpha_j}, \frac{2}{\alpha_j} + I_{n \neq 0} \right)$$

(3.18)

and

$$x_j = \frac{u_n T B_l}{(1+K_{l,j})B_j + u_n T B_l}$$

(3.19)

Notice that the incomplete Beta function is denoted as $B_x(a, b) = \int_0^x z^{a-1} (1 - z)^{b-1} dz$ for $0 \leq x \leq 1$, and $w_n$ and $u_n$ are derived by (3.10) for Rician factor $K_l$. The zero/non-zero indicator functions of $n$ are as defined in Lemma 3.1.

**Proof:** Provided in Appendix A1.3.

Rayleigh fading can be seen as a special case of Rician fading when factor $K_{l,j} = 0$. The simplified analytical results in Corollary 3.1 are achieved in this case by setting $K_{l,j} = 0$ in Proposition 3.2 and the hypergeometric representations as shown in (3.15) and (3.16) of the incomplete Beta function [152].

**Corollary 3.1:** When the desired signal from the serving BS in the $l^{th}$ tier experiences Rician fading with factor $K_l$, and the interfering signals from the tier $j$ experience Rayleigh fading, the coverage probability of a considered user at the distance $r_l$ is
\[ p_{c,l}(T|\eta_l) = \sum_{n=1}^{N} w_n e^{-\pi \sum_{j=1}^{2} \frac{\lambda_j \overline{P}_j}{\alpha_j} \left( \lambda_j \overline{P}_j \left( u_n T, \alpha_j, \overline{B}_j \right) \right)^\frac{2}{\alpha_j} r_l^2 } \]  

where \( w_n \) and \( u_n \) are derived by (3.10) for Rician factor \( K_l \), and

\[ Z(u_n T, \alpha_j, \overline{B}_j) = \frac{2 u_n T \overline{P}_j}{\alpha_j - 2} \left( \frac{2}{\alpha_j - 2} - \frac{2}{\alpha_j} \right) \]  

By integrating the location-dependent performance provided in Corollary 3.1 over \( f_{UE}(\eta_l) \), the PDF of user locations from its serving BS derived in [20], the tier coverage probability is

\[ p_{c,l}(T) = \int_0^{\infty} p_{c,l}(T|\eta_l) f_{UE}(\eta_l) d\eta_l \]

(a) \[ = \int_0^{\infty} p_{c,l}(T|\eta_l) \frac{2\pi \lambda_l}{A_l} \frac{r_1}{A_l} e^{-\pi \sum_{j=1}^{2} \lambda_j \left( \frac{\overline{P}_j}{\alpha_j} \right)^{2/\alpha_j} r_l^2 } dr_l \]

(b) \[ = \frac{\pi \lambda_l}{A_l} \sum_{n=1}^{N} w_n 2 r_l e^{-\pi \sum_{j=1}^{2} \lambda_j \left( \frac{\overline{P}_j}{\alpha_j} \right)^{2/\alpha_j} \frac{2}{\alpha_j} r_l^2 } \]

(c) \[ = \frac{\pi \lambda_l}{A_l} \sum_{n=1}^{N} w_n \int_0^{\infty} e^{-\pi \sum_{j=1}^{2} \lambda_j \left( \frac{\overline{P}_j}{\alpha_j} \right)^{2/\alpha_j} \frac{2}{\alpha_j} r_l^2 } \frac{1}{\alpha_j} dv \]

where \( A_l \) in (a) denotes the probability of a user associating to the tier \( l \) whose formula is given in (3.23), the result in (3.20) is applied in (b), and (c) employs a change of variables \( v = r_l^2 \) and also exchanges the order of the summation and the integration.

\[ A_l = 2\pi \lambda_l \int_0^{\infty} r_l \exp \left( -\pi \sum_{j=1}^{2} \lambda_j \left( \frac{\overline{P}_j}{\alpha_j} \right)^{2/\alpha_j} r_l^2 / \alpha_j \right) dr_l \]

(3.23)

When all the tiers have the same exponent of the log-distance path loss, \( \alpha_j = \alpha \) for any \( j = 1,2 \), thus \( \alpha_j = 1 \), the tier coverage probability in (3.22) is simplified to

\[ p_{c,l}(T) = \sum_{n=1}^{N} w_n \frac{\pi \lambda_l}{A_l} \int_0^{\infty} e^{-\pi \sum_{j=1}^{2} \lambda_j \left( \frac{\overline{P}_j}{\alpha_j} \right)^{2/\alpha_j} Z(u_n T, \alpha_j, \overline{B}_j)^2 \frac{2}{\alpha_j} } dv \]
\[ A_l^{-1} = \sum_{n=1}^{N} w_n \left( \sum_{j=1}^{2} \hat{\lambda}_j \hat{B}_j^{\alpha j} \right)^2 \left[ Z(u_n T, \alpha_j, \hat{B}_j) \right]^{-1} \]  

(3.24)

where \( Z(u_n T, \alpha_j, \hat{B}_j) \) is defined by (3.21) and

\[ A_l^{-1} = \sum_{j=1}^{L \geq 2} \hat{\lambda}_j \hat{B}_j^{\alpha j} \]  

(3.25)

The coverage probability is achieved for a homogeneous network by setting \( \lambda_2 = 0 \), stated in the following Corollary 3.2.

**Corollary 3.2:** When the desired signal from the serving BS in a PPP-modeled homogeneous cellular network experiences Rician fading with factor \( K \), and the interfering signals from other BSs experience Rayleigh fading, the coverage probability of a considered user at the distance \( r \) from its serving BS is

\[ p_c(T, r) = \sum_{n=1}^{N} w_n \frac{1}{\pi \lambda_j Z(u_n T, \alpha, 1) r^2} \]  

(3.26)

And the network coverage probability is

\[ p_c(T) = \sum_{n=1}^{N} w_n \left[ Z(u_n T, \alpha, 1) + 1 \right]^{-1} \]  

(3.27)

where \( w_n \) and \( u_n \) are derived by (3.10) for Rician factor \( K \), and \( Z(u_n T, \alpha, 1) \) is defined in (3.21).

**3.3.4 Spectral efficiency**

**Proposition 3.3:** If the coverage probability for the desired user at the distance \( r \) from its serving BS is known, then the location-dependent spectral efficiency for the user at the distance \( r \) is calculated by:

\[ \tau_l(r) = \int_{T>0} \frac{1}{T+G} p_{c,l}(T|r) dT \]  

(3.28)

where \( G \) is the SIR gap from Shannon capacity and \( p_{c,l}(T|r) \) is given by (3.17). Similarly, if the tier coverage probability is known, then the tier spectral efficiency is
\[ \tau_l = \int_{T > 0} \frac{1}{T+G} p_{c,l}(T) dT \]  

(3.29)

where \( p_{c,l}(T) \) is given by (3.22).

**Proof:** Provided in Appendix A1.4.

By Proposition 3.3, the impact of the SIR gap \( G \) on spectral efficiency is straightforward. With an increase in \( G \), \( \frac{1}{T+G} \) decreases monotonically, so do \( \tau(r) \) and \( \tau \) due to \( Z > 0 \) and \( p_c(Z) \geq 0 \). Note that Proposition 3.3 is applicable for environments with any type of shadowing and fading. When \( G = 1 \) for an ideal situation, Proposition 3.3 is consistent with the previous result in [145].

**Table 3.1 Assumed System Parameter Values [21] [67]**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Assumed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda = [\lambda_1, \lambda_2] )</td>
<td>BS intensity of tiers 1 and 2</td>
<td>([1, 10]\times 1.28\text{E}-6/m^2)</td>
</tr>
<tr>
<td>( P = [P_1, P_2] )</td>
<td>Transmit power of tiers 1 and 2</td>
<td>([46, 26])dBm</td>
</tr>
<tr>
<td>( \alpha = [\alpha_1, \alpha_2] )</td>
<td>Path loss exponent of tiers 1 and 2</td>
<td>([3.76, 3.76])</td>
</tr>
<tr>
<td>( B = [B_1, B_2] )</td>
<td>Association Bias of tiers 1 and 2</td>
<td>([1, 2])</td>
</tr>
<tr>
<td>( \sigma = [\sigma_1, \sigma_2] )</td>
<td>Standard Deviation of Shadowing in tiers</td>
<td>([0, 0]) dB</td>
</tr>
<tr>
<td>( K = [K_1, K_2] )</td>
<td>Rician factor of tiers for the desired signal</td>
<td>(K_1=0, K_2 =1, 5, 10)</td>
</tr>
<tr>
<td>( K_l = [K_{11} \ K_{12} \ K_{21} \ K_{22}] )</td>
<td>Rician factor ( K_{ij} ) for the interfering signals from tier ( j ) to tier ( l )</td>
<td>([0 \ 0] )</td>
</tr>
<tr>
<td>( \rho = [\rho_1, \rho_2] )</td>
<td>Frequency reuse factors of tiers 1 and 2</td>
<td>([1, 1])</td>
</tr>
<tr>
<td>( \eta = [\eta_1, \eta_2] )</td>
<td>Transmission probability by BSs in tiers 1 and 2</td>
<td>([1,1])</td>
</tr>
<tr>
<td>( G )</td>
<td>SIR gap from Shannon capacity</td>
<td>3dB</td>
</tr>
</tbody>
</table>
This subsection presents numerical results to demonstrate the usefulness of the proposed exponential-series approximation and the derived analytic result for coverage probability. A 2-tier HetNet comprising the first tier of macro BSs and the second tier of small BSs is assumed for illustration. The results for the small BSs tier are presented for Rician factor $K = 1, 5$ and $10$. Results for Rayleigh fading channel, denoted by $K = 0$, are also shown for completeness. Except noted otherwise, the assumed system parameter values shown in Table 3.1, mainly borrowed from [21] [67], are used in this subsection, and the subscripts “1” and “2” denote the macro BS tier and the small BS tier, respectively.

### Table 3.2 Coefficients of Exponential-series approximation for Rician Fading

<table>
<thead>
<tr>
<th>Term Index</th>
<th>$K = 1$</th>
<th>$K = 5$</th>
<th>$K = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_n$</td>
<td>$u_n$</td>
<td>$w_n$</td>
</tr>
<tr>
<td>$n = 1$</td>
<td>-0.8993</td>
<td>1.2475</td>
<td>42.253</td>
</tr>
<tr>
<td>$n = 2$</td>
<td>5.9324</td>
<td>1.4298</td>
<td>-189.99</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>-5.4477</td>
<td>1.7436</td>
<td>192.97</td>
</tr>
<tr>
<td>$n = 4$</td>
<td>1.4145</td>
<td>2.0326</td>
<td>-44.229</td>
</tr>
</tbody>
</table>

### A. Validation of the proposed approximation

Assuming 4 terms in (3.6) (i.e., $N = 4$), the optimization problem in (3.10) is solved to determine the values of the coefficients $w_n$ and $u_n$. Table 3.2 lists the calculated coefficients and Fig. 3.2 shows a plot of $\bar{F}(x)$ and $\bar{G}(x)$, the exact and approximate CCDF, respectively. As shown in Fig. 3.2, the closeness between the approximate and exact CCDFs indicates good accuracy of the proposed exponential-series approximation.
Fig. 3.2 CCDF of the desired signal power under Rician fading

B. Validation of the analytical results by simulation

All the analytical results presented in this subsection, shown by continuous curves, are verified by the good agreement with simulation results shown by marks. In the simulation, instead of the unlimited Euclidean plane as assumed in analysis, a circular service area of 50km radius is considered and simulation results are collected with the considered user at the center of the circular area. Other aspects follow the system models described in Section 3.2, without the approximation used in the analysis presented in Section 3.3. The basic steps of the simulation include:

(B1) Locations of BSs in both tiers are uniformly distributed over the circular service area of 50km radius.

(B2) The considered user at the center of the circular area associates to the tier based on the biased association policy (3.3) which is stated in (A6).
The received power of the desired signal from the associated BS is calculated which experiences log-distance path loss and Rician fading, as stated in (A4) with corresponding parameters setting \((\sigma = [0,0]\, \text{dB})\).

All the BSs in each tier are randomly allocated into frequency groups and transmission slots due to frequency reuse and transmission probability.

The received power of the interfering signals experiencing log-distance path loss and Rician fading (if \(K_I \neq 0\)) or Rayleigh fading (if \(K_I = 0\)) are collected from BSs in the same frequency group and transmission slot as that allocated to the serving BS of the considered user.

The loop from (B1) to (B5) runs \(10^7\) times to collect the values of the 4-tuple consisting of the index of the tier the considered user associates to, the distance to the associated BS, corresponding SIR and spectral efficiency. After that, simulation results of the coverage probabilities and spectral efficiencies are attained by histogramming the collected data.

C. **Performance impact of Rician fading**

The Rician/Rayleigh fading situation is investigated first where the coverage probability at distance \(r_2 = 150\, \text{m}\) and the 2\(^{\text{nd}}\) tier coverage probability \((l = 2)\) are shown in Figs. 3.3(a) and 3.3(b), respectively each parameterized by the Rician factors \(K = 5\) and \(K = 10\) for the desired signal. Notice that all the interfering signals are assumed to be Rayleigh faded in the Rician/Rayleigh situation [68] which means \(K_I = 0\). Compared to the results with the assumption of Rayleigh fading \((K = 0)\) for the desired signal, both the location-dependent and the 2\(^{\text{nd}}\) tier coverage probabilities with \(K = 5\) and \(K = 10\) achieve considerable improvement in the lower range of SIR threshold, because the presence of the constant dominant component reduces the probability of severe fading. In particular, if we set the location-dependent coverage probability for the small BSs tier at 90% as
a performance objective for users at the edge area, Fig. 3.3(b) can be used to determine the corresponding SIR threshold. When $K$ increases from 0 to 10, the SIR threshold for 90% coverage probability increases from $-3.3 \text{dB}$ to $+1.3 \text{dB}$, a significant improvement. Furthermore, it is seen from Fig. 3.3(b) that using the Rayleigh fading model for the desired signal provides a conservative value of location-dependent coverage probability, at an SIR threshold less than approximately $+5 \text{dB}$.

At the same time, if the HetNet is designed to operate outside of this range, the type of fading model assumed is immaterial for the tier coverage probability.

![Fig. 3.3 Impact of Rician fading on coverage probability under Rician/Rayleigh fading](image)

(a) Coverage probability at $r_2 = 150 \text{m}$  
(b) Tier 2 Coverage probability

Notice that the location-dependent coverage probability at the high SIR threshold becomes lower with the increase of Rician factor, as shown in Fig. 3.3(a), due to reduced power variation. However, after integrating over the coverage area of a small BS, the coverage probability over the whole range of considered SIR threshold is improved with the increase of Rician factor, as shown in Fig. 3.3(b).
The reason is, for example, at a location $r_A$ from a BS, Rician fading gives a lower coverage probability for a high SIR threshold, for example, $T_0$, compared to Rayleigh fading. However, for a location $r_B$ close to the BS enough, $r_B < r_A$, Rician fading gives a higher coverage probability for this high SIR threshold $T_0$, since the path loss is reduced and the coverage probability moves to the right on the whole. Thus, by integrating the location-dependent coverage probability over the whole area covered by this BS, Rician fading always has a higher coverage probability compared to Rayleigh fading.

![Graphs showing location-dependent spectral efficiency and distance distribution](image)

Fig. 3.4 Impact of Rician fading on spectral efficiency and distance distribution from a user to serving BS in each tier

The location-dependent spectral efficiency are shown in Fig. 3.4(a) for the Rician factor $K = 0$, $K = 5$ and $K = 10$. Compared to the results with Rayleigh fading ($K = 0$), both the spectral efficiencies with $K = 5$ and $K = 10$ achieve marginal improvement, since the spectral efficiency is
mainly determined by the average SIR which is very close to each other for different value of Rician factor. In particular, the spectral efficiency improves by 13% (3.647bits/s/Hz vs 4.125bits/s/Hz) at \( r = 100m \) when \( K \) increases from 0 to 10, not as significant as the aforementioned SIR threshold enhancement from \(-3.3\text{dB}\) to \(+1.3\text{dB}\) for the 90% coverage probability requirement. For users far away from their serving BSs (\( r_2 > 300 \)), the difference among them even becomes indistinguishable. To acknowledge the significant distance range for the location-dependent performance, the distance distribution from a user to serving BS in the tiers \( l = 1 \) and \( l = 2 \) is also shown in Fig. 3.4(b).

Now the Rician/Rician fading situation for the tier of small BSs is considered where the interference from the tier of small BSs is also assumed to be Rician faded with \( K_I = 10 \), in addition to the Rician faded desired signal with \( K = 10 \). The situation is reasonable since the interference is dominated by its component from neighboring small BSs, whose distance to the considered user is not far away with possibility of LOS existence in some situations, especially for dense deployment of small BSs. As shown in Fig. 3.5, the coverage probability at \( r = 100 \) in this Rician/Rician situation is almost the same as that in the Rician/Rayleigh situation where \( K = 10 \) but \( K_I = 0 \) in the tier of small BSs. Notice that this observation holds for the small BS to macro BS intensity ratio \( \lambda_2/\lambda_1 \leq 10 \), in this case, the interference from macro BSs are still Rayleigh faded, at the same time, the interference from small BSs varies due to location uncertainty of small BSs. For the same coverage probability, both Rician/Rician and Rician/Rayleigh fading situations give almost the same spectral efficiency, 1.39bits/s/Hz for the Rician/Rician situation, and 1.41bits/s/Hz for the Rician/Rayleigh situation. These results imply that, with the existence of the interference from macro BSs under Rayleigh fading, the assumption of Rayleigh fading for interference has provided enough accuracy to evaluate
the coverage probability and spectral efficiency, even for some situations where the interference from small BSs is dominated by the LOS component.

Both the coverage probability and spectral efficiency for small BSs in the Rician/Rician situation are slightly lower than those in the Rician/Rayleigh situation, since the function of SIR is convex in terms of the interference from the tier of small BSs (the second derivative of signal to interference ratio with respect to the interference is positive). The interference from small BSs is more widely distributed around the average value in the Rician/Rayleigh situation, while the interference from small BSs in the Rician/Rician situation is dominant with the deterministic component of LOS near to the average value. Thus the SIR under Rician/Rayleigh situation seen as the average value of the convex function is higher than the SIR under Rician/Rician situation approximately seen as the convex function of the average value.
D. Performance impact of reuse factor and transmission probability with Rician fading

![Graphs showing 2nd tier coverage probability and 2nd tier location-dependent spectral efficiency](image)

Fig. 3.6 Frequency reuse impact on coverage probability and spectral efficiency in the small BSs tier of HetNets (Results for Rayleigh fading are shown for comparison with that for Rician fading.)

The impacts of both the reuse factor $\rho_j$ and transmission probability $\eta_j$ in the tier $j$ of PPP-modeled HetNets are characterized as a PPP thinning with the intensity of $\lambda_j$ formulated in (3.11). Thus, the increase of reuse factor in the random pattern has the same effect on though interference mitigating and performance enhancement as the decrease in transmission probability. For example, the increase of $\rho = [1, 1]$ to $\rho = [3, 3]$ mitigates the interference impact on the coverage probability and special efficiency, completely equivalent to the decrease of the full transmission $\eta = [1, 1]$ to $\eta = [1/3, 1/3]$, and both actually reduce to one third of original density of the interfering BSs. Only the results with the increase of reuse factor $\rho = [1, 1]$ to $\rho = [3, 3]$ are shown in Fig. 3.6(a) for the tier
coverage probability and in Fig. 3.6(b) for spectral efficiency, but notice that these discussions are also applicable to the decrease of the transmission probability.

SIR threshold is assumed to be $T = -4dB$ which results in the block error rate of LTE physical downlink control channel of less 1% [28]. As shown in Fig. 3.6(a), for reuse factor $\rho = [3, 3]$, the 2nd tier coverage probability at $T = -4dB$ is 95%, provides significant improvement compared to 75% for the universal frequency reuse $\rho = [1, 1]$. Correspondingly, the (link) spectral efficiency rises by 109% at $r = 200m$ (from 1.69bits/s/Hz to 3.54bits/s/Hz) as shown in Fig. 3.6(b).

![System Spectral Efficiency vs Distance](image)

Fig. 3.7 Impact of frequency reuse on system spectral efficiency in the small BS tier of HetNets

However, only 1/3 of the total bandwidth is available for the considered user since frequency reuse, leading to the system spectral efficiency, defined as the ratio of the link spectral efficiency over the reuse factor and shown in Fig. 3.7, is lowered by 30% at $r = 200m$ (from 1.69bits/s/Hz to 1.18bits/s/Hz). This indicates that in HetNets the increase of the reuse factor improves the coverage probability significantly, however, with the cost of a reduced spectral efficiency at the system level,
especially for users close to the serving small cells. At the same time, for users far away from its serving small cells (such as \( r_2 \geq 250m \)), the coverage probability improves significantly, but with a cost of high reduction in the system spectral efficiency. This observation also holds in the tier of macro BSs and inspires some advanced location-dependent interference management strategies, such as the fractional frequency reuse (FFR) discussed in Chapter 5.

3.4 Performance Impact of Shadowing in 2-tier HetNets with Rayleigh Fading

In this section, the Laplace transform of the interference with lognormal shadowed Rayleigh fading is derived first, then followed by a study of the performance impact of lognormal shadowing. The Gauss-Hermite quadrature in numerical analysis [66] is exploited at both steps in order to circumvent the extra layers of integration due to the analytical challenge of lognormal shadowing.

3.4.1 Laplace transform of tier interference with lognormal shadowed Rayleigh fading

**Lemma 3.2:** When the positions of the interfering BSs (in the tier \( j \)) are modeled by a PPP \( \phi_j \) with the intensity of \( \lambda_j \) in an unlimited Euclidean plane excluding the area \( b(0, d_j) \), the interference free disc of radius \( d_j \) around the considered user, and all the interfering signals are Lognormal shadowed and Rayleigh faded, the Laplace transform of the aggregate interference \( I_j \) received by the considered user is

\[
\mathcal{L}_{I_j}(s) = e^{-\sqrt{\pi s} \lambda_j P_j} \sum_{n=1}^{N} \omega_n \frac{2^{\alpha_j} \exp(\sqrt{\sigma_j} a_n)}{(\alpha_j - 2)} {}_2F_1\left[1, 1 - \frac{2}{\alpha_j}, 2, -s P \sigma_j^\alpha_{d_j} \exp(\sqrt{\sigma_j} a_n)\right]
\] (3.30)

where the weights \( \omega_n \) and the abscissas \( a_n \) are predetermined parameters by the Hermite polynomial [66].

**Proof:** Provided in Appendix A1.5.
3.4.2 Coverage probability in lognormal shadowed Rayleigh fading environment

In this subsection, first the coverage probability in Lemma 3.3 is conditioned with the known value of shadowing $\mu^{-1}$ for desired signal, in this way, the desired signal is Rayleigh faded thus the coverage probability is expressed with the derived Laplace transform of the interference. After that, the coverage probability unconditioned in terms of shadowing $\mu^{-1}$ is derived in Proposition 3.4, where the Gauss-Hermite quadrature in numerical analysis [66] is again exploited. Notice that $\mu^{-1}$ instead of $\mu$ is used since in this case the desired signal power is exponential distributed with the rate parameter $\mu$, which is similar with $u_{n}$ in Section 3.3, and based on the results in (A1.7.2), the CCDF of the desired signal power can be expanded to the form of exponential series after using the Gauss-Hermite quadrature.

Lemma 3.3: When the desired signal experiences Rayleigh fading with the conditioned value $\mu^{-1}$ of shadowing, and the interfering signals experience lognormal shadowed Rayleigh fading, the coverage probability for the considered user associating to the tier $l$ at the distance $r_l$ is

$$p_{c,l}(T|\mu^{-1}, r_l) = e^{-\sqrt{\pi} \sum_{j=1}^{2} \lambda_j \bar{I}_j \sum_{n=1}^{N} \omega_n Z(\mu T \exp(\sqrt{2} \sigma_j a_n), \alpha_j, \bar{B}_j) r_l^{\frac{2}{\alpha_j}}}$$

(3.31)

where

$$Z(x, \alpha_j, \bar{B}_j) = \frac{2 x \bar{B}_j^{\alpha_j^{-1}}}{\alpha_j^{\alpha_j-2}} 2F_1 \left(1,1 - \frac{2}{\alpha_j}; 2 - \frac{2}{\alpha_j}; -\frac{x}{\bar{B}_j} \right)$$

(3.32)

In (3.31) $N$ is the order of Gauss-Hermite integration, the weights $\omega_n$ and the abscissas $a_n$ are predetermined parameters by Hermite polynomial.

Proof: Provided in Appendix A1.6.

About (3.31), numerical results in Section 3.4.3 indicate that small value of $N (N = 8)$ is sufficient to provide acceptable accuracy for the coverage probability and spectral efficiency. The next step
considers the unconditioned case where the lognormal shadowing of the desired signal is also unknown, then Proposition 3.4 follows.

**Proposition 3.4:** When both the desired signal and the interfering signals experience lognormal shadowed Rayleigh fading, the coverage probability for the considered user at the distance \( r \) from its serving BS in the tier \( l \) is

\[
p_{c,l}(T|r) = \sum_{m=1}^{M} \frac{\omega_m}{\sqrt{\pi}} e^{-\sqrt{\pi} \sum_{j=1}^{2} \lambda_j \beta_j^{m}} \sum_{n=1}^{N} \omega_n Z(\exp(\sqrt{2} \sigma_j \alpha_n - \sqrt{2} \sigma_l \alpha_m) \tau, \alpha_j, \beta_j) r_i^{2} \tag{3.33}
\]

where \( Z(x, \alpha_j, \beta_j) \) is defined in (3.32).

**Proof:** Provided in Appendix A1.7.

Numerical results in Section 3.4.3 indicate that both the orders of Gauss-Hermite integrations \( M = N = 8 \) in (3.33) is sufficient for acceptable accuracy.

Finally, the spectral efficiency is then derived by similar steps in Section 3.3.4, hence not repeated here for brevity of presentation.

### 3.4.3 Numerical results and discussion

Except where noted, the assumed values of parameters used in the evaluations in this section are shown in Table 3.3, where channel parameters are mainly borrowed from simulation baseline parameters in [21], and the spectral efficiency is expressed in the unit of bits/s/Hz. Different from Section 3.3.5 that focused on the tier of small BSs where Rician fading more likely happens, the results for the first tier of macro BSs are presented here and the results for the tier of small BSs are omitted for brevity since both tier presents the same trends in terms of the impact of shadowing.

All the analytical results presented in this subsection, shown by curves, are verified by simulations, shown by marks. In the simulations, instead of the unlimited Euclidean plane as assumed in analysis,
a circular area (radius of 50km) is considered and simulation results are collected only from the considered user in the BS closest to the center of the circular area. Other aspects follow the system models in Section 3.2 but without any approximation used in the analysis in Section 3.4. The details of the simulation follow the depiction given in Section 3.3.5.B, except the steps (B3) and (B5) with some differences outlined as follows.

(B3) The received power of the desired signal $h$ from the associated BS is calculated which experiences log-distance path loss, shadowing, and Rayleigh fading, as stated in (A4) with corresponding parameters setting ($K = 0$).

(B5) The received power of the interfering signals experiencing log-distance path loss, shadowing, and Rayleigh fading ($K = 0$) is collected from BSs in the same frequency group and transmission slot as that allocated to the serving BS of the considered user.

Table 3.3 Assumed System Parameter Values [21] [28]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Assumed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = [\lambda_1, \lambda_2]$</td>
<td>BS intensity of tiers</td>
<td>$[1, 10]*1.28E{-6}/m^2$</td>
</tr>
<tr>
<td>$R_M$</td>
<td>Apothem of hexagonal tessellation leading to the same BS density as $\lambda_1$</td>
<td>1500m</td>
</tr>
<tr>
<td>$P = [P_1, P_2]$</td>
<td>Transmit power of tiers</td>
<td>$[46, 26]$dBm</td>
</tr>
<tr>
<td>$\alpha = [\alpha_1, \alpha_2]$</td>
<td>Path loss exponent of tiers</td>
<td>$[3.76, 3.76]$</td>
</tr>
<tr>
<td>$B = [B_1, B_2]$</td>
<td>Association Bias of tiers</td>
<td>$[1, 2]$</td>
</tr>
<tr>
<td>$\sigma = [\sigma_1, \sigma_2]$</td>
<td>Standard Deviation of Shadowing</td>
<td>$[8, 8]$ in dB</td>
</tr>
<tr>
<td>$K = [K_1, K_2]$</td>
<td>Rician factor of tiers for the desired signal</td>
<td>$[0, 0]$</td>
</tr>
<tr>
<td>$K_I = \begin{bmatrix} K_{11} &amp; K_{12} \ K_{21} &amp; K_{22} \end{bmatrix}$</td>
<td>Rician factor $K_{ij}$ for the interfering signals from tier $j$ to tier $l$</td>
<td>$[0 \ 0]$</td>
</tr>
<tr>
<td>$\eta = [\eta_1, \eta_2]$</td>
<td>Transmission probabilities of tiers</td>
<td>$[1, 1]$</td>
</tr>
<tr>
<td>$\rho = [\rho_1, \rho_2]$</td>
<td>Frequency reuse factors of tiers</td>
<td>$[1, 1]$</td>
</tr>
<tr>
<td>$G$</td>
<td>SIR gap</td>
<td>3dB</td>
</tr>
</tbody>
</table>
A. Impact of shadowing on coverage probability and spectral efficiency

The impact of shadowing on the location-dependent coverage probability and spectral efficiency is shown in Figs. 3.8(a) and 3.8(b) for the tier of macro BSs, parameterizing with the shadowing standard deviation of $\sigma_{dB}$ in the range from 0dB to 12dB. Similarly, as mentioned in [17] for homogeneous cellular networks, the impact of shadowing on coverage probability and spectral efficiency in HetNets is marginal for small value of shadowing standard deviation, for example, $\sigma_{dB} = 4dB$. Shown in Figs. 3.8(a) and 3.8(b), at a relative distance $r/R_M = 0.5$, the coverage probability at $T = -4dB$ only decreases from 86% to 78% and the 1st tier spectral efficiency decreases from 1.37bits/s/Hz to 1.29bits/s/Hz, when $\sigma_{dB}$ increases from 0dB to 4dB. However, when $\sigma_{dB}$ continues to increase from 4dB to 12dB, the coverage probability at $r/R_M = 0.5$
decreases from 78% to 41% and the spectral efficiency at $r/R = 0.5$ decreases by 35% from 1.29bits/s/Hz to 0.84bits/s/Hz, which shows the significant impact of shadowing on the performance when shadowing becomes severe, consistent with the conclusion in [97]. Note that, because of the different association assumptions as discussed in Section 3.2, the impact of shadowing in this section is different from that in [80] [81] [82], where the results indicate that the coverage probability and the spectral efficiency are not affected by arbitrary shadowing in an interference limited network.

![Fig. 3.9 Tier coverage probability and spectral efficiency comparison for different frequency reuse factor settings under lognormal shadowed Rayleigh fading](image)

(a) Tier coverage probability at $T = -4dB$  
(b) Tier spectral efficiency

The tier coverage probability and spectral efficiency results are presented in Fig. 3.9 for the macro BS tier of the HetNet with different shadowing standard deviation. It is seen from Fig. 3.9 that, at frequency reuse factors $\rho = [1,1]$ and $\rho = [3,3]$, the tier coverage probability and spectral efficiency monotonically decrease with an increase in the shadowing standard deviation, due to the strong shadowing worsening the received SIR. For $K = 1$, the tier coverage probability at $T = -4dB$, shown in Fig. 3.8(a), decreases from 64% to 35% when $\sigma_{dB}$ increases from 3dB to 12dB.
Correspondingly, the tier spectral efficiency shown in Fig. 3.9(b) decreases by 34% (i.e. from 1.80 bits/s/Hz to 1.18 bits/s/Hz) for the macro BS tier. Clearly from Figs. 3.8 and 3.9, the shadowing significantly deteriorates both the spectral efficiency and coverage probability when the biased association policy is exploited.

B. Performance impact of reuse factor and transmission probability under lognormal shadowed Rayleigh fading

As mentioned in Section 3.3.5, the increase of reuse factor from $\rho = [1,1]$ to $\rho = [3,3]$ has the same performance impact as the decrease of the transmission probability from the full load $\eta = [1,1]$ to partial load $\eta = [1/3,1/3]$ . Thus, only the performance impact of decreasing transmission probability is discussed here, as shown in Fig. 3.10, but note that, these discussions are also applicable to the increase in reuse factor. With a decrease of transmission probability from $\eta = [1,1]$
to $\eta = [1/9, 1/9]$, the coverage probability at $T = -4 dB$ for users at $r/R_M = 0.5$ increases from 59% to 92% in Fig. 3.10(a), because of the decreasing intensity of interferers resulting in reduced interference on the whole. For the same reason, the spectral efficiency at $r/R_M = 0.5$ increases by 381% from 1.08bits/s/Hz to 4.12bits/s/Hz in Figs. 3.10(b), provided that the BS the considered user associates with is transmitting. At the system level, however, the spectral efficiency decrease by 57% from 1.08bits/s/Hz to 0.46bits/s/Hz, since only one ninth of frequency-time resource is utilized for partial load situation where $\eta = [1/9, 1/9]$ is assumed.

3.5 Summary

This chapter has developed the analytical approach for performance evaluation of PPP-modeled HetNets under Rician fading propagation situation and the lognormal shadowed Rayleigh fading environment. In addition to analytical results of coverage probability and spectral efficiency, there are some other contributions presented. First, the finite exponential-series approximation to the non-central chi-squared distribution has been proposed, and it converges to the exact value with the increase of terms. Numerical results show the computation efficiency of the exponential–series approximation in that only 4 terms of the exponential-series provide acceptable accuracy to approximate the power distribution of Rician fading with factor $K \leq 10$. Notice that the power of the shadowed Rayleigh fading is actually represented in the form of exponential series too, after exploiting the Gauss-Hermite quadrature. Second, the Laplace transform of the aggregate interference from a PPP-modeled tier under both the Rician fading situation and the lognormal shadowed Rayleigh fading environment is derived. Based on these, the performance impact of LOS existence and shadowing is evaluated, respectively. Numerical results show that, in HetNets with the biased association policy, the shadowing deteriorates both the coverage probability and spectral
efficiency significantly, especially in the range of the standard shadowing deviation $\sigma_{\text{dB}} > 4\text{dB}$. The performance impact of Rician fading is a little complicated. For the desired signal, the existence of LOS has a significant enhancement for coverage probability when the SIR threshold is less than 5dB, though the resulted improvement of spectral efficiency is marginal. For the interfering signals, however, the impact of LOS existence can be completely ignored on both the coverage probability and spectral efficiency when the small BS to macro BS intensity ratio $\lambda_2/\lambda_1 \leq 10$. Third, some key system parameters, such as frequency reuse, power allocation among tiers, and transmission probability, are considered in the proposed approach. Numerical results indicate that the partitioning of frequency resource in PPP-modeled HetNets will alleviate the impact of intra-tier and inter-tier interference, however, with the cost on the spectral efficiency at the system level. Conversely, the universal frequency reuse provides the best system spectral efficiency in PPP-modeled HetNets with the penalty on the coverage probability. An in-depth look on location-dependent performance metrics reveals that the spectral efficiency loss due to high frequency reuse factor is marginal for a user far away from its serving BS, which provides the reasoning for location-dependent frequency reuse interference management strategies, such as FFR (topic of Chapter 5), to satisfy the stringent requirements on communications reliability and efficiency.
Chapter 4: Performance Analysis of Hybrid-Modeled HetNets with Hexagonal Tessellated Macro Base Stations over Fading Channels

4.1 Introduction

Performance analysis of HetNets comprising a hybrid model [98] where the locations of macro BSs are well planned and the small BSs are randomly positioned is investigated in this chapter. Inspired by the previous works on stochastic geometry [17] [20] and the fluid model [130] [153], this chapter provides an analytical approach to evaluate the performance of hybrid-modeled HetNets with hexagonal tessellated macro BSs and randomly located small BSs. First, the hybrid model is presented which combines together location regularity of the well-planned macro BSs and topological randomness of small BSs. For this hybrid model, analytical results on the coverage probability and spectral efficiency are derived under several different fading and shadowing situations. Second, the performance impact of key system parameters such as frequency reuse factor and transmission probability on performance is characterized successfully, and the impact of increasing intensity of small BSs on both metrics is investigated in detail, which provides some key insights on the network planning significance in terms of the increasing intensity of small BSs. Third, we are interested in the performance at user location in terms of the nearest macro BS [154]. In addition to the aggregate performance metrics for the whole network or each tier, location-dependent metrics are derived which facilitate the evaluation of location-dependent resource management strategies and metrics, and performance fairness among users at different locations is also investigated.

The remainder of this chapter is organized as follows. The system model is presented in Section 4.2. Analyses of the coverage probability and spectral efficiency are derived under different propagation
situations in Section 4.3, including Rayleigh fading, Rician/Rayleigh fading, and the lognormal shadowed Rayleigh fading. Section 4.4 presents the verified numerical results by which the performance impacts of Rician fading, lognormal shadowing, reuse factor and transmission probability, and increasing intensity of small BSs, are evaluated and discussed. After that, the similarities and differences among the analyses for PPP-modeled HetNets and hybrid-modeled HetNets are demonstrated in Section 4.5, followed by the summary in Section 4.6.

### 4.2 System Model

A downlink hybrid-modeled HetNet with 2 tiers of BSs in the unlimited Euclidean plane is considered in this chapter, characterized by the assumptions (A1) to (A7) stated in Section 3.2 of Chapter 3 but now with modifications on the assumptions (A1) and (A2) to consider the hexagonal tessellated macro BSs, stated as follows:

(A1) *BS deployment:* Instead of the assumption that both tiers are generated by a homogeneous PPP in PPP-modeled HetNet, here the first tier of the hybrid-modeled HetNet is assumed to be formed by

![Fig. 4.1 A hybrid-modeled HetNet with hexagonal tessellated macro BSs and randomly positioned small BSs](image-url)
well-planned macro BSs $\phi_1 = \{\phi_{1,i}, i = 1, 2, \cdots \}$ where $\phi_{1,i}$ is the location of macro BS $i$. Notice that the set of macro BSs locations $\phi_1$ does not form a PPP. Specifically, as shown in Fig. 4.1, each BS of the first tier $\phi_1$ is located at the center of a hexagon with the apothem of $R_M$. Conversely, the second tier comprises randomly-dropped small BSs $\phi_2 = \{\phi_{2,i}, i = 1, 2, \cdots \}$ where $\phi_{2,i}$ is the location of small BS $i$ in tier 2, in an unlimited Euclidean plane, and $\phi_2$ is assumed to be modeled as a PPP with the intensity $\lambda_2$. For convenience, the average number of macro BSs per unit area, $\lambda_1 = 1/(2\sqrt{3}R_M^2)$, is defined in this chapter, which does not require that the macro BSs be randomly located.

Fig. 4.2 Hexagonal geometry of macro BSs with frequency reuse in the regular pattern

(A2) Spectrum allocation: In the PPP-modeled HetNet, the frequency groups in both tiers are assumed to be randomly and independently allocated. Instead, the $\rho_2$ frequency groups in only the second tier of the hybrid model are assumed to be randomly allocated, since the small BSs are randomly located as the PPP $\phi_2$ and a regular pattern is not possible in a random deployment [14] [17]. However in the first tier of the hybrid model, macro BSs are hexagonal tessellated and the $\rho_1$
frequency groups are allocated as the regular pattern of frequency reuse [97], as shown in Fig. 4.2 where $\rho_1 = 3$ is assumed for demonstration where cells in green house co-channel macro BSs. This type of regular patterns provides the maximum distance to the neighboring co-channel macro BSs where the frequency reuse factor $\rho_1$ is the size of the frequency group cluster and the frequency reuse distance $D_0 = 2R_M\sqrt{\rho_1}$ [87].

4.3 Performance Analysis of 2-Tier HetNets over Fading Channels

In this section, the interference approximation is proposed first in 4.3.1 for the tier of macro BSs, and Lemma 4.1 formulates the probability of the user at the distance $r_1$ associating to each tier. Based on these results, the coverage probability is derived in Proposition 4.1 for Rayleigh fading, then the analytical results are extended to the Rician/Rayleigh fading situation in Proposition 4.2 and the lognormal shadowed Rayleigh fading situation in Proposition 4.3. The analytical results for the homogeneous cellular network with hexagonal tessellation, as a special case of the hybrid model, are also presented with lower computational complexity, at the end of this section.

4.3.1 Approximation of interference statistics from hexagonal tessellated macro BSs

The performance analysis involving hexagonal tessellation is challenging because the locations of macro BSs are determined on a regular lattice [17]. The fluid model in [153] approximates the interference from hexagonal cellular networks as being generated by interfering BSs that are randomly located outside of a disc, but without consideration of the impact of the propagation phenomena such as shadowing and fading. In order to capture the statistics of the interference from hexagonal tessellation, this chapter proposes an approach by using the tool of stochastic geometry. Similar to the fluid model, this chapter approximates the aggregate interference from the hexagonal tessellation as being generated by the fictitious interfering BSs which are randomly located in the
shaded area in Fig. 4.3 and modeled as a homogeneous spatial PPP, instead of being generated by
the interfering BSs located at the center of hexagons as shown in Fig. 4.2. This approximation is
made because the overall interference is dominated by the contribution of the nearest co-channel
BSs and the impact of their distance is embodied by the radius \( d \) of the interference free disc in the
proposed approach. The foregoing is confirmed as a good approximation in Section 4.4.1.

As shown in Fig. 4.3, the considered user is at a distance \( r_1 \) from its closest macro BS and a minimal
distance \( D = 2R_M\sqrt{\rho_1} - r_1 \) from the ring of the first tier of interfering BSs. The interfering BSs for
the approximate interference are randomly located away from the user at a minimum distance \( d = \delta \times D \),
where \( \delta \) is the reuse distance scale factor lying between 0 and 1, i.e. \( \delta \in [0,1] \).

\[
d = \delta \times \left(2R_M\sqrt{\rho_1} - r_1\right)
\]  

(4.1)

Fig. 4.3 Approximate the interference from macro BSs as being generated by fictitious BSs
which are randomly located outside of a disc as a spatial PPP

The radius \( d \) of the interference free disc decreases with an increase in \( r_1 \) which means that the
interference becomes location-dependent, and users at a large value of \( r_1 \) experience worse
interference. The reuse distance scale factor \( \delta \) is introduced to define the lower boundary of the
region of the interfering BSs which are distributed randomly. The smaller the scale factor, the larger
the interference region resulting in an increase in the number of BSs which are near the considered
user and, consequently, increased interference to the user of interest. Numerical results show that
the value $\delta = 0.77$ gives interference from macro BSs at random locations (as shown in Fig. 4.3)
that closely approximates the interference generated by all the interfering BSs located at the center
of hexagons (Fig. 4.2). The area density of the BSs in Fig. 4.2 is $\lambda_1^{BS} = 1/(2\sqrt{3}R_M^2)$, and there are
only $(1/\rho_1)$th of BSs (being co-channel) each with the independent transmission probability $\eta_1$
interfering with the considered user, resulting in the area density of the effective interferers from the
first tier $j = 1$ being

$$\lambda_j = \lambda_j^{BS}\eta_j/\rho_j$$  \hspace{1cm} (4.2)

Even in terms of the analysis of homogeneous networks comprised of hexagonal tessellated macro
BSs, the distance scale factor introduced and the analytical results derived in this chapter for
homogeneous networks including the aforementioned Laplace transform of the interference are
completely new and different from the previous work on the fluid model. Without consideration of
shadowing and fading, the fluid model only gives overestimated spectral efficiency and there are no
results on coverage probability. The numerical results on spectral efficiency based on the proposed
approach in this chapter are compared with those of the fluid model in Section 4.4.7.

Move to the second tier $j = 2$ of randomly deployed small BSs. With joint consideration of the
frequency reuse factor $\rho_j$ and transmission probability $\eta_j$, the formulation of (4.2) is obviously
applicable for the tier $j = 2$.

The analytical results of the Laplace transform of the aggregate interference in Section 3.3.2 and
Section 3.4.1, formulated as (3.16) for Rayleigh, and (3.30) for lognormal shadowed Rayleigh
fading, are thus applicable to the tier of macro BSs and the tier of small BSs in the hybrid model. Notice that the discussion on Laplace transform of the aggregate interference under Rician fading is ignored in this chapter, since the existence of LOS in the interference does not significantly impact the coverage probability and thus spectral efficiency, as demonstrated in Section 3.3.5.

4.3.2 Probability of considered user at \( r_1 \) associating to each tier

**Lemma 4.1**: the probability of a user associating to the first tier is

\[
Pr\{l = 1\} = \int_0^{R_2^2} e^{-\lambda^{BS}_2 \pi \left(\frac{B_2 P_2}{B_1 P_1}\right)^\frac{2}{\alpha_2} r^{\frac{\alpha_1}{\alpha_2}} dr} \quad (4.3)
\]

When \( \alpha_1 = \alpha_2 = \alpha \), a closed-form result is given as follows,

\[
Pr\{l = 1\} = \frac{1-e^{-c}}{c} \quad (4.4)
\]

where

\[
c = \frac{\lambda^{BS}_2}{\lambda^{BS}_1} \left(\frac{B_2 P_2}{B_1 P_1}\right)^\frac{2}{\alpha} \quad (4.5)
\]

The probability of a user associating to the second tier follows as

\[
Pr\{l = 2\} = 1 - Pr\{l = 1\} = \int_0^{R_2^2} e^{-\lambda^{BS}_2 \pi \left(\frac{B_2 P_2}{B_1 P_1}\right)^\frac{2}{\alpha_2} r^{\frac{\alpha_1}{\alpha_2}} dr} \quad (4.6)
\]

**Proof**: Provided in Appendix A2.1.

4.3.3 Coverage probability of 2-tier hybrid-modeled HetNets

**Proposition 4.1**: When the desired signal and the interfering signals experience Rayleigh fading, the coverage probability for a user at \( r_1 \) associating to the first tier is

\[
p_{c,l=1}(T|r_1) = e^{-\sum_{j=1}^{2} y_{ij}(r_1)} \quad (4.7)
\]

And the coverage probability for a user at \( r_1 \) associating to the second tier is
\[ p_{c_{l=2}}(T|r_1) = \int_0^{R_2} \left[ 1 - \frac{n_1}{\rho_1(1+T_{r_1})} \right] e^{-\sum_{j=1}^2 v_{lj}(r_2)} f_{r_2}(r_2) dr_2 \]  

(4.8)

where in (4.7) and (4.8), \( T \) is the SIR threshold,

\[ \nu_{lj}(r_1) = \frac{2\pi\lambda_l d_{lj}^{2-a_j} T_{r_1}^{a_l p_j}}{\rho_l^{(a_j-2)}} 2F_1 \left[ 1, 1 - \frac{2}{a_j}; 2 - \frac{2}{a_j}; -\frac{T_{r_1}^a p_j}{r_{lj}^{a_j}} \right] \]  

(4.9)

\[ f_{r_2}(r_2) = \frac{2\pi\lambda_2 r_2 e^{(-\lambda_2\pi r_2^2)}}{1-\exp(-\lambda_2\pi R_2^2)}, 0 < r_2 \leq R_2 \]  

(4.10)

In (4.9), the notation \( 2F_1 [\cdot;\cdot;\cdot] \) is the Gauss-Hypergeometric function, the subscript \( l \) denotes the associated tier index and \( j \) the tier index for the interference. \( d_{lj} \) is the radius of the interference-free disc in terms of the interference from tier \( j \) for users associating to tier \( l \), specifically, \( d_{11} = d_{21} = 0.77 \times (2R - r_1) \), \( d_{12} = R_2 \), and \( d_{22} = r_2 \).

**Proof:** Provided in Appendix A2.2.

The analysis in Proposition 4.1 is extended as follows to consider the Rician/Rayleigh fading situation in Proposition 4.2 by combining the similar logic for Proposition 3.2, and the lognormal shadowed Rayleigh fading environment in Proposition 4.3 by combining the similar logic for Proposition 3.4.

**Proposition 4.2:** When the desired signal is Rician faded with factor \( K_l \) and the interfering signals experience Rayleigh fading, then the coverage probability of the first tier for a user at the distance \( r_1 \) from the closest macro BS is

\[ p_{c_{l=1}}(T|r_1) = \sum_{n=1}^N w_n e^{-\sum_{j=1}^2 \nu_{lj}(r_1, \mu_n)} \]  

(4.11)

And the coverage probability of the second tier for a user at the distance \( r_1 \) from the closest macro BS is
\[ p_{c,l=2}(T|r_1) = \sum_{n=1}^{N} w_n \int_{0}^{R_2} \left[ 1 - \frac{\eta_1}{\rho_1(\mu_n T - 1)} \right] e^{-\Sigma_{j=1}^{2} v_{ij}(r_2, \mu_n) f_{r_2}(r_2) dr_2} \]  \tag{4.12}

where in (4.11) and (4.12), \( w_n \) and \( u_n \) are derived by (3.10) for Rician factor \( K_l \), \( f_{r_2}(r_2) \) is defined in (4.10), and

\[ v_{ij}(r_1, \mu_n) = \frac{2\pi d_{ij}^{2-\alpha_j} \mu_n T r_i^{\alpha_i} p_j}{p_i(\alpha_j-2)} 2F_1 \left[ 1, 1 - \frac{2}{\alpha_j}; 2 - \frac{2}{\alpha_j}; -\frac{\mu_n T r_i^{\alpha_i} p_j}{p_i d_{ij}^{\alpha_j}} \right] \]  \tag{4.13}

**Proof**: Provided in Appendix A2.3.

Next consider the lognormal shadowed Rayleigh fading situation, the coverage probability is derived in Proposition 4.3, by combining the derivation in Proposition 4.1 and the Gauss-Hermite quadrature [66] as exploited in Lemma 3.2 and Proposition 3.4. The proof is omitted for brevity of presentation.

**Proposition 4.3**: When the desired signal and the interfering signals experience lognormal shadowed Rayleigh fading, the coverage probability for a user at \( r_1 \) associating to the first tier is

\[ p_{c,l=1}(T|r_1) = \sum_{m=1}^{M} \frac{\omega_m}{\sqrt{\pi}} e^{-\Sigma_{j=1}^{2} \frac{\omega_n}{\sqrt{\pi}} v_{ij}(\sqrt{2} \sigma_j a_n - \sqrt{2} \sigma_i a_m)} \]  \tag{4.14}

And the coverage probability for a user at \( r_1 \) associating to the second tier is

\[ p_{c,l=2}(T|r_1) = \sum_{m=1}^{M} \frac{\omega_m}{\sqrt{\pi}} \int_{0}^{R_2} \left[ 1 - \frac{\eta_1}{\rho_1(1+e^{\sqrt{2} \sigma_j a_n - \sqrt{2} \sigma_i a_m})} \right] e^{-\Sigma_{j=1}^{2} \frac{\omega_n}{\sqrt{\pi}} v_{ij}(\sqrt{2} \sigma_j a_n - \sqrt{2} \sigma_i a_m)} f_{r_2}(r_2) dr_2 \]  \tag{4.15}

where in (4.14) and (4.15), \( f_{r_2}(r_2) \) is defined in (4.10), and \( v_{ij}(r_1, \mu_n) \) in (4.13)

By combining the results in Lemma 4.1 with each tier’s location-dependent coverage probability in Propositions 4.1 to 4.3, the coverage probability for a user at \( r_1 \) (which probably associates to tier 1 or tier 2) is calculated by the Law of total probability [141],

89
\[ p_c(T|r_1) = \sum_{l=1}^{2} p_{c,l}(T|r_1) \cdot Pr\{l|r_1\} \quad (4.16) \]

where \( p_{c,l}(T|r_1) \) is derived in (4.7) and (4.8) for the tiers \( l = 1 \) and \( l = 2 \), respectively, under Rayleigh fading; (4.11) and (4.12) under Rician/Rayleigh fading; (4.14) and (4.15) under the lognormal shadowed Rayleigh fading. \( Pr\{l = 1|r_1\} \) is given in (A2.1.2) and \( Pr\{l = 2|r_1\} = 1 - Pr\{l = 1|r_1\} \).

Integrating the aforementioned results of the coverage probability at distance \( r_1 \) to the closest macro BS over the corresponding pdf of \( r_1 \) gives the tier coverage probabilities for the tier \( l = 1 \) in (4.17), \( l = 2 \) in (4.18), and the network coverage probability in (4.19), respectively.

\[
\begin{align*}
p_{c,t=1}(T) &= \int_0^{R_1} p_{c,t=1}(T|r_1)f_{r_1}(r_1|l = 1)dr_1 \\
p_{c,t=2}(T) &= \int_0^{R_1} p_{c,t=2}(T|r_1)f_{r_1}(r_1|l = 2)dr_1 \\
p_c(T) &= \int_0^{R_1} p_c(T|r_1)f_{r_1}(r_1)dr_1
\end{align*}
\quad (4.17, 4.18, 4.19)
\]

where \( p_{c,t=1}(T|r_1) \) and \( p_{c,t=2}(T|r_1) \) are given in Proposition 4.1 to 4.3 under different propagation situations, and \( f_{r_1}(r_1|l = 1) \) can be derived from (A2.1.5), \( f_{r_1}(r_1|l = 2) \) from (A2.1.6), \( p_c(T|r_1) \) is formulated in (4.16), and \( f_{r_1}(r_1) \) in (A2.1.3).

### 4.3.4 Coverage probability of homogeneous networks comprising hexagonal tessellated macro BSs

Finally, as a special case of hybrid-modeled HetNets, a homogeneous cellular network with hexagonal tessellation is considered where the simplified analytical results are achieved. By setting \( \lambda_2 = 0 \) which means that there are no small BSs existing and the network consists of only macro BSs with hexagonal tessellation, the coverage probabilities are formulated by the following corollaries 4.1 to 4.3 for different propagation situations, derived from Propositions 4.1 to 4.3, respectively.
Corollary 4.1: When the desired signal and the interfering signals experience Rayleigh fading, the coverage probability for a user at the distance $r_1$ from the serving macro BS is

$$p_c(T|r_1) = e^{-\frac{2\pi\lambda_1 d^2 - \alpha_1 \mu_n T r_1^{a_1}}{a_1 - 2} - \frac{\alpha_1}{a_1} - \mu_n T d^{-\alpha_1 r_1^{a_1}}} \, 2F_1\left[1,1 - \frac{2}{a_1}, \frac{2}{a_1}; -\mu_n T d^{-\alpha_1 r_1^{a_1}}\right]$$

(4.20)

The network coverage probability is derived by integrating (4.20) over the distribution of $r_1$

$$p_c(T) = \int_0^{R_1} \frac{2\pi\lambda_1 d^2 - \alpha_1 \mu_n T r_1^{a_1}}{a_1 - 2} - \frac{\alpha_1}{a_1} - \mu_n T d^{-\alpha_1 r_1^{a_1}} \, dr_1$$

(4.21)

where $2F_1[; ; ]$ is the Gauss-Hypergeometric function and $d$ denotes the radius of the interference-free disc for the considered user at $r_1$, $d = 0.77 \times (2R - r_1)$.

Corollary 4.2: When the desired signal is Rician faded with factor $K_1$ and the interfering signals experience Rayleigh fading, then the coverage probability for a user at the distance $r_1$ from the serving macro BS is

$$p_c(T|r_1) = \sum_{n=1}^{N} w_n e^{-\frac{2\pi\lambda_1 d^2 - \alpha_1 \mu_n T r_1^{a_1}}{a_1 - 2} - \frac{\alpha_1}{a_1} - \mu_n T d^{-\alpha_1 r_1^{a_1}}} \, 2F_1\left[1,1 - \frac{2}{a_1}, \frac{2}{a_1}; -\mu_n T d^{-\alpha_1 r_1^{a_1}}\right]$$

(4.22)

The network coverage probability is derived by integrating (4.22) over the distribution of $r_1$

$$p_c(T) = \sum_{n=1}^{N} w_n \int_0^{R_1} \frac{2\pi\lambda_1 d^2 - \alpha_1 \mu_n T r_1^{a_1}}{a_1 - 2} - \frac{\alpha_1}{a_1} - \mu_n T d^{-\alpha_1 r_1^{a_1}} \, dr_1$$

(4.23)

where in (4.22) and (4.23) $w_n$ and $u_n$ are derived by (3.10) for Rician factor $K_1$.

Corollary 4.3: When the desired signal and the interfering signals experience lognormal shadowed Rayleigh fading, the coverage probability for a user at the distance $r_1$ from the serving macro BS is

$$p_c(T|r_1) = \sum_{m=1}^{M} \frac{\omega_m}{\sqrt{\pi}} e^{-\frac{2\pi\lambda_1 d^2 - \alpha_1 \mu_n T r_1^{a_1}}{a_1 - 2} - \frac{\alpha_1}{a_1} - \mu_n T d^{-\alpha_1 r_1^{a_1}}} \, 2F_1\left[1,1 - \frac{2}{a_1}, \frac{2}{a_1}; -\mu_n T d^{-\alpha_1 r_1^{a_1}}\right]$$

(4.24)
The network coverage probability is derived by integrating (4.24) over the distribution of $r_1$ (A2.1.3),

$$p_c(T) = \sum_{m=1}^M \frac{\omega_m}{\sqrt{\pi}} \int_0^{R_1} \frac{2r_1}{R_1^2} e^{\frac{2\sqrt{\pi} \lambda_1 T r_1^d}{d^2 \sigma_1^2 - 2(\sigma_1 - \gamma)}} \sum_{n=1}^N \omega_n e^{\frac{\sqrt{\pi} \sigma_1 (a_n - a_m)}{d^2 \sigma_1}} \frac{\sqrt{\pi} \sigma_1 (a_n - a_m) T r_1^d}{d^2 \sigma_1} \frac{1}{2} \frac{2}{2 - \frac{2}{\sigma_1}} e^{\frac{\sqrt{\pi} \sigma_1 (a_n - a_m) T r_1^d}{d^2 \sigma_1}} \frac{1}{2} \frac{2}{2 - \frac{2}{\sigma_1}} e^{\frac{\sqrt{\pi} \sigma_1 (a_n - a_m) T r_1^d}{d^2 \sigma_1}} d r_1$$

(4.25)

In (4.24) and (4.25), $\omega_n$ and $a_n$ are predetermined parameters by Hermite polynomial, and the numerical results in Section 4.4 show that the numbers of Hermite polynomial terms $M = N = 8$ gives acceptable accuracy.

### 4.3.5 Spectral efficiency

The spectral efficiency $\tau$ is derived as another key metric for evaluating the hybrid-modeled HetNet performance. Note that the spectral efficiency derived for the PPP-modeled HetNet in Chapter 3 is also applicable to the hybrid model, which is evaluated in Section 4.4 and included here for completeness.

**Proposition 4.4:** For a user at $r_1$ to the closest macro BS, the location-dependent spectral efficiency (in nats/s/Hz) of tier $l$ at $r_1$ is

$$\tau_l(r_1) = \int_{T > 0} \frac{1}{T + G} p_{c,l}(T|r_1) dT$$

(4.26)

and the tier corresponding spectral efficiency is

$$\tau_l = \int_{T > 0} \frac{1}{T + G} p_{c,l}(T) dT$$

(4.27)

where $p_{c,l}(T|r_1)$ and $p_{c,l}(T)$ are given by (4.14) and (4.17) for $l = 1$, (4.15) and (4.18) for $l = 2$, respectively.
Table 4.1 Assumed System Parameter Values [21] [28]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Assumed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_M$</td>
<td>Apothem of hexagon</td>
<td>1500m ($\lambda_1 = 0.128$/km$^2$)</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Intensity of small BSs</td>
<td>10 $\times$ 0.128/km$^2$</td>
</tr>
<tr>
<td>$P = [P_1, P_2]$</td>
<td>Transmit power of tiers 1 and 2</td>
<td>[46, 26]dBm</td>
</tr>
<tr>
<td>$\alpha = [\alpha_1, \alpha_2]$</td>
<td>Path loss exponent of tiers 1 and 2</td>
<td>[3.76, 3.76]</td>
</tr>
<tr>
<td>$B = [B_1, B_2]$</td>
<td>Association bias of tiers 1 and 2</td>
<td>[1, 2]</td>
</tr>
<tr>
<td>$\sigma = [\sigma_1, \sigma_2]$</td>
<td>Standard deviation of shadowing in tiers</td>
<td>[0, 0]dB</td>
</tr>
<tr>
<td>$K = [K_1, K_2]$</td>
<td>Rician factor for the desired signal in tiers</td>
<td>[0, 0]</td>
</tr>
<tr>
<td>$K_{ij} = \begin{bmatrix} K_{11} &amp; K_{12} \ K_{21} &amp; K_{22} \end{bmatrix}$</td>
<td>Rician factor $K_{ij}$ for the interfering signals from tier $j$ to tier $l$</td>
<td>$\begin{bmatrix} 0 &amp; 0 \ 0 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>$\rho = [\rho_1, \rho_2]$</td>
<td>Frequency reuse factor in tiers 1 and 2</td>
<td>[1,1]</td>
</tr>
<tr>
<td>$\eta = [\eta_1, \eta_2]$</td>
<td>Transmission probability in tiers 1 and 2</td>
<td>[1,1]</td>
</tr>
<tr>
<td>$G$</td>
<td>SINR gap from Shannon capacity</td>
<td>3dB</td>
</tr>
</tbody>
</table>

4.4 Numerical Results and Discussion

This section presents numerical results to demonstrate the usefulness of the derived analytical results and to discuss the performance trend of a hybrid 2-tier heterogeneous cellular network with increasing small BS to macro BS intensity ratio, in addition to the impact of Rician fading and shadowing. Except noted, the assumed values of parameters used in the numerical evaluation are shown in Table 4.1, where the channel parameters are mainly borrowed from [21] and [28], and the subscripts “1” and “2” denote the macro BS tier and the small BS tier, respectively. The range of inter-site distance among macro BSs varies depending on the radio access system and the network scenario considered, and the apothem of hexagon in this thesis is set to be $R_M=1500$m which is in the range considered in LTE [28] [22]. Actually, under interference limited situation, the performance metrics at $r_1$ considered in this thesis actually depend on the relative distance $r_1/R_M$, 

93
instead of $R_M$. That is, the location-dependent performance results will remain unchanged with $r_1/R_M$, even if $R_M$ is changed, for example, $R_M = 500$m. For this reason, the location-dependent performance results are presented generally parameterized with the relative distance $r_1/R_M$ in this thesis.

**4.4.1 Validating proposed analysis through simulation results**

Deterministic locations of base stations in large hexagonal cellular networks make exact analysis intractable [63], hence this chapter proposes an analytical approach with the key approximation in Section 4.3.1: the aggregate interference in hexagonal cellular networks is approximated as being generated by fictitious interfering BSs randomly located outside of a disc. The accuracy of this key approximation is validated by Monte Carlo simulations in Matlab [155] where the key approximation is relaxed (i.e. without the approximation made in Section 4.3). The steps of the simulation are outlined as follows.

(B1) Each center of hexagonal area has one macro BS in hexagonal tessellation over a circular service area of 50km radius, overlaid by small BSs which are uniformly and randomly dropped.

(B2) The considered user is uniformly located in the central hexagonal area, and it associates to the tier based on the biased association policy (3.3) which is stated in (A6).

(B3) The received power of the desired signal $h$ from the associated BS is calculated which experiences log-distance path loss, shadowing, and Rician fading, as stated in (A4) in Section 3.2 with corresponding parameters setting in this section: $\sigma = 0$ when the shadowing is not considered; and $K = 0$ when the fading is Rayleigh instead of Rician.
The macro BSs are allocated into frequency groups \( \rho_1 \) following the regular pattern while the small BSs are randomly allocated into frequency groups \( \rho_2 \). After that, transmission slots are allocated to BSs in each tier with the corresponding transmission probability \( \eta = [\eta_1, \eta_2] \).

The received power of the interfering signals experiencing log-distance path loss, shadowing (were \( \sigma = 0 \) if no shadowing), and Rician fading (i.e., \( K_f \neq 0 \)) or Rayleigh fading (i.e. \( K_f = 0 \)) are collected from BSs in the same frequency group and transmission slot as that allocated to the serving BS of the considered user.

The loop from (B1) to (B5) runs \( 10^7 \) times to collect the values of the 4-tuple consisting of the index of the tier the considered user associates to, the distance to the associated BS, corresponding SIR and spectral efficiency. After that, simulation results of the coverage probabilities and spectral efficiencies are attained by histograms.

The simulation results are represented by marks and the analytical results by curves in Fig. 4.4. Probability density function of SIR is compared between the proposed approach (by the negative derivative of coverage probability) and Monte Carlo simulation, parameterized by the shadowing standard deviation \( \sigma_{dB} \) in Fig. 4.4(a) and by the Rician factor \( K \) in Fig. 4.4(b). The coverage probability in the macro BSs with the setting of reuse factor \( \rho = [3,3] \) is presented in Fig. 4.4(c), parameterized by the user distance from its serving base station. Fig. 4.4(d) gives the location-dependent spectral efficiency for different reuse factors. All the results of the proposed approach with different settings in Fig 4.4 are almost overlapped with the corresponding results from simulation, thus confirming the accuracy of the analytical results. Simulation results are also presented by marks in the following figures up to Fig 4.9, for further validation of the proposed approach under different scenarios.
Fig. 4.4 Validation of analytical results by simulation results for hybrid-modeled HetNets

(a) SIR Probability density function at $r_1/R_M = 0.5$ in 1st tier

(b) SIR Probability density function in 2nd tier

(c) Location-dependent coverage probability in 1st tier with $\rho = [3,3]$

(d) Location-dependent spectral efficiency in 1st tier with different settings of frequency reuse
4.4.2 Coverage probability trend with densifying small BSs

Analytical results of coverage probability at distance $r_1 = 750m = 0.5R_M$ are presented in Fig. 4.5(a) for the first tier $p_{c,l=1}(T|r_1)$ and Fig. 4.5(b) for the second tier $p_{c,l=2}(T|r_1)$. Notice that the lack of association bias, i.e. $B = [1,1]$ from Section 4.4.2 to Section 4.4.3, in order to focus on the impact of hexagonal tessellated macro BSs. With increasing small BS to macro BS intensity ratio $\lambda_2/\lambda_1$, the user at $r_1$ associating to the first tier experiences deteriorating interference from small BSs which continually lowers the coverage probability. As shown in Fig. 4.5(a), for example, the coverage probability at $r_1/R_M = 0.5$ decreases from 97% to 43% for the desired SIR threshold $T = -4dB$ when the small BS to macro BS intensity ratio increases from 1 to 100, which indicates that the coverage area of macro BSs shrinks with the increase of intensity ratio $\lambda_2/\lambda_1$, due to the deteriorating interference from small BSs. This phenomenon is also demonstrated by the distance distribution for users in the first tier, shown in Fig. 4.5(d), which shows that, when $\lambda_2/\lambda_1$ increases from 1 to 100, the considered user associating to the first tier locates closer and closer to its serving macro BS with high probability. In order to make the whole service area be covered well by macro BSs, some dedicated resource should be reserved to macro BSs in HetNets with the deployment of dense small BSs.

We next study the performance impact of macro BSs with hexagonal tessellation on the tier of small BSs. When the intensity of small BSs is comparable to or less than that of macro BSs, the distribution of distance from the user to the closest small BS $r_2$ in (4.10) is significantly limited to a smaller area compared to the distribution without this constraint, which leads to the improved coverage probability for the second tier, as shown in Fig. 4.5(b). However, with the increasing intensity of small BSs, the truncated Weibull distribution (4.10) approaches to an un-truncated
Weibull distribution and the distance constraint on $r_2$ due to the biased association policy has marginal impact on it. Meanwhile, the interference from the first tier is fixed but the interference from small BSs becomes more and more dominant. Both factors push the second tier coverage probability close to a lower limit, the coverage probability of a Poisson point process, which is indistinguishable with that in the second tier when $\lambda_2/\lambda_1 = 100$, and it is not shown in Fig. 4.5(b) for conciseness.

The performance impact of macro BSs with hexagonal tessellation is also shown in Fig. 4.5(c). Note that the coverage probability here is not conditioned on the distance $r_1$ any more, different from those in Fig. 4.5(a). As shown in Fig. 4.5(c), the coverage probability of both tiers is less at higher intensity of small BSs, with both approaching the same lower limit, the same as that in Fig. 4.5(b), since the interference for both tiers is dominated by that from the small BSs when the intensity of small BSs is high. Recall that, as mentioned previously, the first tier coverage probability at $r_1$ continuously decreases to zero (instead of the lower bound) with increasing intensity of small BSs. But why does the aggregate coverage probability of the first tier, unconditioned on $r_1$, approach the lower bound, instead of zero, with increasing intensity of small BSs? The reason is, most of first tier users move closer to the serving macro BS, as shown in Fig. 4.5(d), where the increasing power of the desired signal due to reduced distance improves the coverage probability. In this way the impact of the aforementioned worsened coverage probability at $r_1$ is partially balanced. For PPP-based HetNets, when the association biasing factors and path loss exponents are identical among tiers ($B_1 = B_2$ and $\alpha_1 = \alpha_2$), the tier aggregate coverage probability is independent of the tier intensity ratio, and is identical to that in a homogeneous PPP.
4.4.3 Spectral efficiency and fairness with densifying small BSs

Each tier spectral efficiency depends on the location of the considered user in terms of the distance $r_1$ to the closest macro BS, as shown in Fig. 4.6(a) for the first tier, $\tau_1(r_1)$ and Fig. 4.6(b) for the second tier, $\tau_2(r_1)$. When the intensity of small BSs is comparable to that of macro BSs, the impact of the interference from the second tier on $\tau_1(r_1)$ is marginal, which results in $\tau_1(r_1)$ being very close to the spectral efficiency of a hexagonal tessellated homogeneous network, as shown in [79]. However, $\tau_1(r_1)$ continuously decreases to zero with the increasing intensity of small BSs, similar to the behavior exhibited by $p_{c, l=2}(T|r_1)$, as shown in Fig. 4.5(a), due to the increasing interference from the small BS tier.

Shown in Fig. 4.6(b), an interesting phenomenon exists on the results for $\tau_2(r_1)$ which, at small values of $r_1$, achieves a significant improvement compared to that for a homogeneous random network [17] [20], depicted by the dashed line, while $\tau_2(r_1)$ near the edge of hexagon $r_1 = R_M$ approaches to the dashed line. For example, for users at the middle of the hexagon $r_1/R_M = 0.5$ and $\lambda_2/\lambda_1 = 1$, if the users associate to the second tier, they will achieve the spectral efficiency $\tau_2(r_1) = 2.48$bits/s/Hz, much higher than 1.43bits/s/Hz shown in the dashed line. At the same time, users at $r_1/R_M = 1$ have the spectral efficiency $\tau_2(r_1) = 1.56$bits/s/Hz, very close to 1.43bits/s/Hz. The reason is also similar to that for Fig. 4.5(b). For users at $r_1/R_M = 0.5$, if they associate to the second tier, the serving small BS has to be much closer compared to that at $r_1/R_M = 1$, as formulated in (A2.1.1), which improves the spectral efficiency $\tau_2(r_1)$. For users at the edge of the hexagon, the interference is dominated by the component from the second tier and the distribution of distance $r_2$ approaches the un-truncated Weibull distribution resulting in the situation similar to that of a homogeneous random network.
(a) Spectral efficiency for users at $r_1$ in 1$^{st}$ tier  
(b) Spectral efficiency for users at $r_1$ in 2$^{nd}$ tier  

c) Tier spectral efficiency with growing $\lambda_2/\lambda_1$  
(d) Jain’s fairness index with growing $\lambda_2/\lambda_1$

Fig. 4.5 Spectral efficiency and fairness performance with increasing intensity of small BSs

By integrating $\tau_l(r_1)$ over $r_1$ for $l = 1$ in (A2.1.5) and $l = 2$ in (A2.1.6), the spectral efficiency of each tier is calculated. Shown in Fig. 4.6(c), both tiers achieve better spectral efficiency approaching to that of a homogeneous hexagonal cellular network when the intensity of small BSs $\lambda_2$ is
comparable to or less than that of macro BSs $\lambda_1$, since the planned macro BSs with high transmission power constitute the dominant component of the interference. However, if $\lambda_2$ increases to $100\lambda_1$ or higher, the second tier of small BSs becomes the dominant tier and the spectral efficiencies of both tiers decrease to the lower limit of 1.43 bits/s/Hz, the lower performance bound as stated in [17] [99].

As a goal, cellular networks intend to provide similar quality of service (QoS) for users located anywhere in the service area, and here the 2-tier HetNet has potential to achieve this goal with increasing intensity of small BSs. To measure the similarity of the spectral efficiency (a QoS metric) achieved by users at different locations in the macro BS, denoted by fairness, the Jain's fairness index [156] is adopted. By definition, $J = E^2(\tau(r_1))/E(\tau^2(r_1))$ where $E(\cdot)$ is the expectation with respect to distance $r_1$. Clearly, $J$ lies between 0 and 1 and has a direct positive relationship with fairness. As shown in Fig. 4.6(d), the index is 0.48 for $\lambda_2/\lambda_1 = 1$, which means that users at different locations $r_1$ will experience dramatically different spectral efficiency, similar as in a homogeneous cellular network. But when the intensity of small BSs increases to $100\lambda_1$ or even more, the index approaches to about 0.9 which indicates that $\tau(r_1)$ is similar for most of the locations $r_1$, implying that most of the users at different locations experience similar quality of service. Indeed, with increasing small BS intensity relative to macro BS intensity, most of the users associate to small BSs where most of these users receive a similar experience, as shown in Fig. 4.6(b).

**4.4.4 Performance impact of Rician fading**

Now, the small BS tier coverage probability is shown in Fig. 4.7(a) and location-dependent coverage probability at $r_1$ in the small BS tier presented in Fig. 4.7(b), respectively with the Rician factors $K = 0$ and $K = 10$. Compared to the results with Rayleigh fading ($K = 0$), both the small BS tier coverage probability (in the lower range of SIR threshold) and location-dependent spectral
efficiency with \( K = 10 \) achieve considerable improvement, because the constant dominant component in the desired signal power reduces the probability of severe fading. In particular, the tier coverage probability for small BSs increases from 67% to 79% at the SIR threshold \( T = -4 \text{dB} \), when the Rician factor increases from 0 to 10. Furthermore, it is seen from Fig. 4.7(a) that using the Rayleigh fading model for the desired signal provides a conservative value of unconditional coverage probability, at an SIR threshold less than approximately +5dB. If the HetNet is designed to operate outside of this range, the type of fading model assumed is immaterial for coverage probability. The spectral efficiency is also improved with the increasing value of Rician factor. As shown in Fig. 4.7(b), when \( K \) increases from 0 to 10, the spectral efficiency for users associating to the small BSs has improved by about 12% at any position \( r_1 \), and thus the tier spectral efficiency is also improved by 12% from 1.34bits/s/Hz to 1.50bits/s/Hz for users associating to the small BSs.

![Graph of 2nd tier coverage probability](image1)

![Graph of spectral efficiency](image2)

(a) 2nd tier coverage probability  
(b) spectral efficiency for users at \( r_1 \) in 2nd tier

Fig. 4.6 Performance impact of Rician fading
4.4.5 Performance impact of shadowing

Numerical results for the joint Rayleigh fading and lognormal shadowing with $\sigma_{dB} = 0$ dB and $\sigma_{dB} = 8$ dB are shown in Fig. 4.8. Recall that the curve for $\sigma_{dB} = 0$ dB depicts the results for Rayleigh fading without shadowing. Clearly, the lognormal shadowing makes a significant impact on the performance metrics, reducing the coverage probability in the lower range of SIR threshold and decreasing the spectral efficiency substantially for users with higher SIR (i.e. closer to the serving BS or with high reuse factor). In Fig. 4.8(b), at a relative distance $r_1/R_M = 0.5$ where users have a higher SIR than when at the cell edge, the spectral efficiency decreases from 1.79 bits/s/Hz to 1.31 bits/s/Hz (a 27% decrease) when $\sigma_{dB}$ increases from 0 dB to 8 dB. On the other hand, near the cell edge $r_1/R_M = 1$, users have a low SIR, and the corresponding increase in $\sigma_{dB}$ results in a low impact on spectral efficiency (a 9% decrease from 0.35 bits/s/Hz to 0.32 bits/s/Hz) because of the
low average SIR at edge. On the whole, the tier spectral efficiency for macro BSs decreases by 23% from 1.89bits/s/Hz to 1.47bits/s/Hz when the standard deviation $\sigma_{dB}$ of shadowing grows from 0dB to 8dB.

4.4.6 Performance impacts of reuse factor and transmission probability

In PPP-modeled HetNets, the frequency reuse groups are randomly allocated to macro BSs, as stated in the assumption (A2) in Chapter 3, resulting in the same performance impact as the decrease of transmission probability, as discussed in Section 3.3.5-D; while the hybrid model allocates frequency reuse groups in the regular pattern, see the assumption (A2) in Section 4.2, which has the significant performance improvement, since the increase of reuse factor reduces the number of co-channel base stations in a specified area, the same as the reduced transmission probability. At the same time, the increased reuse factor with the regular pattern in hexagonal tessellation maximizes the minimal distance among co-channel macro BS, which is not the case for the reduced transmission probability or the frequency reuse with a random pattern.

In order to focus on the investigation of this difference among the reuse factor and transmission probability, the homogeneous network with hexagonal tessellated macro BSs is considered in this subsection. With the increase of reuse factor form $\rho_1 = 1$ to $\rho_1 = 3$, it is seen from Fig. 4.9(a) that the coverage probability for SIR threshold $T_{dB} = -4dB$ increases from 83% to 98%, and in Fig. 4.9(b) the spectral efficiency for users at the distance $r_1/R_M = 1.0$ increases from 0.57bits/s/Hz to 2.32bits/s/Hz.

Instead, if the transmission probability decreases from $\eta_1 = 1$ to $\eta_1 = 1/3$ and reuse factor remains at 1, the area density of interferers also decreases to one third. Then the coverage probability and spectral efficiency, also shown in Figs. 4.9(a) and 4.9(b), respectively, both are improved due to the
reduced number of interferers. The coverage probability for SIR threshold $T_{dB} = -4dB$ increases from 83% to 94% and the spectral efficiency for users at $r_1/R_M = 1.0$ increases from 0.57bits/s/Hz to 1.56bits/s/Hz when $\rho_1 = 1$ and transmission probability is reduced from $\eta_1 = 1$ to $\eta_1 = 1/3$.

Through changing from $\rho_1 = 1$ to $\rho_1 = 3$, or changing from $\eta_1 = 1$ to $\eta_1 = 1/3$, both lead to the same reduction of the area density of interference, while the former gives more improvement on coverage probability and spectral efficiency, as shown in Fig. 4.9. The improvement is because an increase of the reuse factor with the regular pattern has an advantage of maximizing the minimal distance among the co-channel macro BSs, consequence of the increased distance from the nearest interfering BSs: $d = \delta \times (2R_M \sqrt{\rho_1} - r_1)$.

![Graphs showing coverage probability and location-dependent spectral efficiency](image)

(a) Coverage probability  
(b) Location-dependent spectral efficiency

Fig. 4.8 Difference of performance impacts between reuse factor and transmission probability for a homogeneous network with hexagonal tessellated macro BSs
Comparison of proposed approach with fluid model

Fig. 4.9 Spectral efficiencies calculated by the fluid model and the proposed approach in Rayleigh fading situation

The spectral efficiency in a homogeneous network with hexagonal tessellated macro BSs obtained via the fluid model [14] (shown by dashed curves in Fig. 4.10) is compared to that achieved by the proposed approach (depicted by solid curves in Fig. 4.10) in Rayleigh fading environment ($\sigma_1 = 0$).

It is seen that, at a given value of $\rho_1$, the fluid model overestimates the spectral efficiency over the range of the relative distance considered. For example, at the distance $r_1/R_M = 1.0$, the fluid model overestimates the spectral efficiency from the proposed approach by 263% and 186% for $\rho_1 = 1$ and $\rho_1 = 3$, respectively. There are two reasons for the overestimation. First, the fluid model underestimates the interference without introducing the scale factor $\delta = 0.77$. As mentioned in Section 4.3.1, the scale factor needs to be introduced because the neighboring region of the deterministic nearest interfering macro BSs needs to be included in the PPP. Second, without considering signal fading, the fluid model optimistically uses the average SIR instead of the
instantaneous SIR, which further overestimates the spectral efficiency, as pointed out by Jensen’s inequality for the concave modified Shannon’s formula [157].

4.5 Comparison of Hybrid-modeled HetNets with PPP-modeled HetNets

PPP-modeled HetNets are investigated in Chapter 3 and hybrid-modeled HetNets are considered in this chapter. The differences between both models are about the deployment of macro BSs and the frequency reuse patterns: macro BSs are randomly located in the former while macro BSs are hexagonal tessellated in the latter. Correspondingly, the frequency groups are randomly allocated among macro BSs in the former while in the latter the frequency groups are allocated with a well-planned regular pattern. The determined locations of macro BSs on a regular lattice introduces the analytical challenges and requires a new approach, which makes the analyses in this chapter different from that in Chapter 3. In Chapter 3, the performance at the specified distance to its serving BS is derived first and then the tier performance metrics are achieved. Instead, this chapter derives the performance metrics at the distance $r_1$, the distance from the considered user to the closest macro BSs, since the locations of macro BSs are fixed and the distance from the user to the macro BSs is easier to achieve, compared to the randomly deployed small BSs. PPP-modeled HetNets can also be analyzed in the same way as the hybrid-modeled HetNets, by following the main ideas developed in this chapter, but without the interference approximation of macro BSs made in Section 4.3.1 and some differences shown in shaded columns in Tables 4.2 and 4.3. After that, the performance of the two models is compared in Section 4.5.2.
4.5.1 Comparison of analyses - a unified framework with differences in user distribution and interference characteristics

Key analytical results on probability of associating to each tier and the radius of interference-free disc when considering the interference from each tier are listed in the following Table 4.2 and Table 4.3 for hybrid-modeled HetNets and PPP-modeled HetNets, respectively. The differences between Table 4.2 and Table 4.3 are shown in the shaded column, including $d_{11}$ (radius of interference-free disc for the interference from macro BSs to the user associating to the tier of macro BSs), $d_{21}$ (radius of interference-free disc for the interference from macro BSs to the user associating to the tier of small BSs) and the PDF of user distance from the nearest macro BS.

Briefly speaking, there are two different aspects between the analyses of PPP-modeled HetNets and hybrid-modeled HetNets. First, they present the opposite trend from the interference from macro BSs with the increase of user distance $r_1$. In the hexagonal tessellation, the user at the greater distance $r_1$ from its nearest macro BS experiences severer interference since the user becomes closer to the neighbor interfering macro BSs with deterministic locations, formulated by $d_{11} = d_{21} = \delta \times \left( 2R_M \sqrt{K} - r_1 \right)$; while in the PPP-modeled HetNets, if the user has the increasing distance $r_1$ to its nearest macro BS, the interference from macro BSs reduces since the interfering macro BSs is limited to be farther away from the user, $d_{11} = d_{21} = r_1$. Second, the distance distributions of the users to the macro BSs in both types of HetNets are different. In the hexagonal tessellation, the distance of the randomly dropped user to its nearest macro BS must be limited to be less than the farthest vertex point; while in PPP-modeled HetNets, the distance of a user to its nearest macro BS is in the range of $(0, \infty)$. In other words, the results of $d_{11}, d_{21}, \text{ and } f_{r_1}(r_1)$ in Table 4.3 are applied instead of those in Table 4.2 to apply the analyses developed in Section 4.3 to PPP-modeled HetNets.
Table 4.2 Hybrid-modeled HetNets for the typical user at the distance \((r_1, r_2)\) to the nearest Macro BS and to the nearest small BS

<table>
<thead>
<tr>
<th>Radius (d_{ij}) of interference-free disc</th>
<th>Interference from tier (j = 1)</th>
<th>Interference from tier (j = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associating to Macrocells (l = 1) (r_2 &gt; R_2 = \left(\frac{B_2P_2}{B_1P_1}\right)^{\frac{a_1}{a_2}} r_1^{\frac{a_1}{a_2}})</td>
<td>(d_{11} = \delta \times (2R_M\sqrt{\rho_1} - r_1))</td>
<td>(d_{12} = \left(\frac{B_2P_2}{B_1P_1}\right)^{\frac{1}{a_2}} \frac{a_1}{a_2} r_1^{\frac{a_1}{a_2}})</td>
</tr>
<tr>
<td>Probability: (e^{-\frac{B_2}{B_1} \pi \left(\frac{B_2P_2}{B_1P_1}\right)^{\frac{2}{a_2}} r_1^{\frac{2a_1}{a_2}}})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Associating to small cells (l = 2) (r_2 \leq R_2 = \left(\frac{B_2P_2}{B_1P_1}\right)^{\frac{1}{a_2}} r_1^{\frac{a_1}{a_2}})</td>
<td>(d_{21} = \delta \times (2R_M\sqrt{\rho_1} - r_1))</td>
<td>(d_{22} = r_2)</td>
</tr>
<tr>
<td>(interference: nearest macro BS and others outside of the disc of the radius (d_{21}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance distribution</td>
<td>(f_{r_1}(r_1) = 2r_1/R_1^2), (f_{r_1}(r_1, l) = f_{r_1}(r_1)P(l</td>
<td>r_1)), (0 &lt; r_1 \leq R_1) where (\pi R_1^2 = 2\sqrt{3}R_M^2)</td>
</tr>
</tbody>
</table>

Table 4.3 PPP-modeled HetNets for the typical user at the distance \((r_1, r_2)\) to the nearest Macro BS and to the nearest small BS

<table>
<thead>
<tr>
<th>Radius (d_{ij}) of interference-free disc</th>
<th>Interference from tier (j = 1)</th>
<th>Interference from tier (j = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associating to Macrocells (l = 1) (r_2 &gt; R_2 = \left(\frac{B_2P_2}{B_1P_1}\right)^{\frac{a_1}{a_2}} r_1^{\frac{a_1}{a_2}})</td>
<td>(d_{11} = r_1)</td>
<td>(d_{12} = \left(\frac{B_2P_2}{B_1P_1}\right)^{\frac{1}{a_2}} \frac{a_1}{a_2} r_1^{\frac{a_1}{a_2}})</td>
</tr>
<tr>
<td>Probability: (e^{-\frac{B_2}{B_1} \pi \left(\frac{B_2P_2}{B_1P_1}\right)^{\frac{2}{a_2}} r_1^{\frac{2a_1}{a_2}}})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Associating to small cells (l = 2) (r_2 \leq R_2 = \left(\frac{B_2P_2}{B_1P_1}\right)^{\frac{1}{a_2}} r_1^{\frac{a_1}{a_2}})</td>
<td>(d_{21} = r_1)</td>
<td>(d_{22} = r_2)</td>
</tr>
<tr>
<td>(interference: nearest macro BS and others outside of the disc of the radius (d_{21}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance distribution</td>
<td>(f_{r_1}(r_1) = 2\pi \lambda_1^2 r_1 e^{-\lambda_1^2 \pi r_1^2}), (f_{r_1}(r_1, l) = f_{r_1}(r_1)P(l</td>
<td>r_1)), (r_1 \in (0, \infty))</td>
</tr>
</tbody>
</table>
Fig. 4.10 Performance comparison of macro BSs between random deployment and hexagonal tessellation under Rayleigh fading
4.5.2 Performance comparison between PPP-modeled and hybrid-modeled HetNets

The coverage probability and location-dependent spectral efficiency in a homogeneous network with macro BSs are compared in Figs. 4.11(a) and 4.11(b). It is seen that, for both types of models, the tier coverage probability at the same threshold is higher when $\rho_1 = 3$ than when $\rho_1 = 1$, due to reduced interference at $\rho_1 = 3$. It is also seen from Fig. 4.11(a) that, over the range of SIR threshold considered, the difference between the coverage probability for the PPP-modeled HetNet and hybrid-modeled HetNet is significant for $\rho_1 = 1$, because the minimum distance between co-channel macro BSs (that is, the frequency reuse distance $D = 2R_M$) is maximized in hexagonal tessellation, whereas in the PPP-modeled HetNet the distance between co-channel BSs is possibly close to zero.

When $\rho_1 = 3$, the further increase of this minimum distance in hexagonal tessellation to $D = 2\sqrt{3}R_M$ results in the enhancement of this difference, seen in Fig. 4.11(a). These differences indicate that without overlay with small BSs, well-planned macro BSs have considerable advantage over PPP-modeled networks where macro BSs are randomly and independently located.

Fig. 4.11(b) compares the location-dependent metric of spectral efficiency under frequency reuse factors $\rho_1 = 1$ and $\rho_1 = 3$. Though the desired signal has the same statistical characteristics in both models, the interference experienced by the considered user is different, explained in the following.

As discussed in Section 4.3.1, the interference from tessellated macro BSs is outside of the circle with the radius $d^{Hybrid} = \delta \times (2R_M\sqrt{K} - r_1)$, a decreasing function of $r_1$. Instead, $d^{PPP} = r_1$, an increasing function of $r_1$, for PPP-modeled networks. When users are at a distance $r_1 < 0.87R_M$, $d^{Hybrid} > d^{PPP}$ which means users in hexagonal tessellated macro BSs experience less interference from neighboring macro BSs, and so achieve a higher spectral efficiency. Conversely, when $r_1 > $
0.87R_M, there are definitely some strong interfering BSs near to the considered user in tessellated macro BSs and correspondingly \( d^{Hybrid} < d^{PPP} \), resulting in more severe interference from the neighboring macro BSs and lower spectral efficiency than that in PPP modeled networks. The crossover point in Fig. 4.11(b) is at the distance \( r_1 = 0.87R_M \) where \( d^{Hybrid} = r_1 = d^{PPP} \), the point where both model have exactly the same interference statistics (and hence spectral efficiency). When \( \rho_1 > 1 \), the condition \( d^{Hybrid} > r \) always holds for users at any location in hexagonal tessellation, hence resulting in higher spectral efficiency than that in PPP modeled homogeneous networks for users at the same distance, as demonstrated in Fig. 4.11(b) for \( \rho_1 = 3 \) for illustration.

Dropping randomly deployed small BSs into both types of homogeneous networks, one with PPP-modeled macro BSs and the other with hexagonal tessellated macro BSs, the discussion in the previous paragraph still holds, though the performance difference of macro BSs between PPP-modeled HetNets and hybrid-modeled HetNets is reduced, since the introduction of small BSs brings the same additional interference to both models in the previous paragraph, reducing the performance difference between both types of HetNets. The differences and similarities between PPP-modeled HetNets and hybrid-modeled HetNets will further be demonstrated in the next chapter where the interference management strategy of FFR will be studied and evaluated for both types of HetNets.

4.6 Summary

An analytical approach is developed for performance analysis of hybrid-modeled HetNets with tessellated macro BSs, to successfully characterize the coverage probability and the spectral efficiency under different radio propagation situations, including the Rayleigh fading situation, the Rician/Rayleigh fading situation, and the lognormal shadowed Rayleigh fading situation. After that,
the similarities and differences between analyses of PPP-modeled HetNets and hybrid-modeled HetNets are listed and performance differences are demonstrated.

Based on these analyses and comparisons, there are some key conclusions achieved. First, modeling the received interference in hexagonal cellular networks approximately by a PPP is a valid approximation for obtaining tractable analytical results, thus the proposed analytical results are developed in this chapter eliminating the need for time-intensive Monte Carlo simulations. Different from that in PPP-modeled HetNets, the phenomenon that cell edge users experience a more severe interference from the neighboring macro BSs than users near the cell center is captured in the proposed approach. Second, the analyses reveal that, tessellated macro BSs reduce the impact of interference for users associating to macro BSs by the increase of minimal distance of users to interfering transmitters, at the same time, tessellated macro BSs also reduce the impact of interference for users associating to small BS, by limiting the users closer to small BSs. For the same intensity for both tiers under the Rayleigh fading situation, the hybrid model shows 50% improvement in spectral efficiency for the first tier (2.14bit/s/Hz vs 1.43bits/s/Hz), and 30% improvement for the second tier (1.85bits/s/Hz vs 1.43bits/s/Hz), compared to the PPP-modeled HetNet. The hybrid model represents the HetNet with well-planned macro BSs, while the PPP model represents the HetNet with randomly dropped macro BSs. The performance advantages of the hybrid model compared to the PPP model actually indicate the significance of network planning on macro BS locations and frequency reuse in practice, that is, in HetNets, well-planned macro BSs reduce the performance impact of interference for all users including users associating to small BSs, compared to randomly dropped macro BSs. This finding shows the significance of network planning at the initial phase of HetNets and provides useful insights on HetNet performance to HetNet
network architects and designers, when the interference from small BSs is not dominant. The results also show that this performance improvement from hexagonal tessellation of macro BSs decreases with the increasing intensity of small BSs and becomes marginal when there are, on average, about 100 small BSs located in each hexagon. Third, as discussed in Section 4.2, dedicated resources should be reserved for macro BSs in HetNets with dense small BSs, in order to make the whole service area be covered by macro BSs, since dense small BSs shrink the coverage area of macro BSs with shared resources to limited local region. Finally, numerical results show that, when the intensity of small BSs is 100 times larger than that of macro BSs, users at different locations will experience similar coverage probability and spectral efficiency with a high probability. It is concluded that the introduction of small BSs with increasing intensity relative to that of macro BSs improves the fairness among users at different locations in the hexagonal area.
5.1 Introduction

As one of interference coordination techniques [102], fractional frequency reuse (FFR) mitigates the severe interference for users at the edge area and thus improves the coverage probability there. Based on the developed location-dependent performance analyses for Poisson point process (PPP)-modeled and hybrid-modeled HetNets in Chapter 4, this chapter considers FFR exploited by the tier of macro BSs in PPP-modeled and hybrid-modeled HetNets to evaluate its benefit on spectral efficiency at the system level, in addition to its benefit on coverage improvement. Specifically, this chapter first develops an approach to analyze the impact of FFR on spectral efficiency with consideration of different network deployments (PPP-modeled HetNets and hybrid-modeled HetNets) under different radio propagation conditions (Rayleigh fading, Rician/Rayleigh fading, and shadowed Rayleigh fading). Then the optimal partitioning of FFR in terms of spectral efficiency is demonstrated in hybrid-modeled HetNets and the impact of system parameters such as transmission probability, standard deviation of shadowing, the signal to interference ratio (SIR) gap form Shannon capacity, and increasing intensity of small BSs on the optimal FFR partitioning is discussed. By this comprehensive investigation on FFR, the following two questions are answered: 1) In what kinds of deployments and environments can FFR outperform the universal frequency reuse scheme in terms of spectral efficiency in addition to coverage enhancement? 2) If FFR can provide the spectral efficiency advantage in some environment, how to approach the optimal partitioning between the central region and the edge region of FFR in terms of spectral efficiency?
The remainder of this chapter is organized as follows. The system model is presented in Section 5.2, and interference statistics and performance metrics are analyzed in Section 5.3. Section 5.4 presents the verified numerical results in hybrid-modeled and PPP-modeled HetNets, by which the trade-off between the coverage probability and the spectral efficiency is discussed. After that, Section 5.5 studies the optimal partitioning of FFR in hybrid-modeled HetNets in terms of spectral efficiency under different settings of system parameters, to answer the two aforementioned questions. Finally Section 5.6 summarizes this chapter.

5.2 System Model

A downlink two-tier HetNet in the unlimited Euclidean plane with FFR used in the tier of macro BSs is considered in this chapter, characterized by the assumptions as stated in Section 3.2 of Chapter 3, with modifications included here to consider macro BSs with both types of deployments and to introduce the FFR concept.

(A1) BS distribution: Both PPP-modeled HetNet and hybrid-modeled HetNet models are considered and compared in this chapter. In a PPP-modeled HetNet, the locations of BSs in each tier $j$ are modeled by a PPP $\phi_j$ in the unlimited Euclidean plane, as stated in (A1) in Chapter 3. In a hybrid-modeled HetNet, small BSs are randomly dropped but macro BSs are hexagonal tessellated, as stated in (A1) in Chapter 4.

(A2) Spectrum allocation: The total available spectrum of $W$ Hz is shared by both tiers., $\text{FFR}(\rho_1^C, \rho_1^E)$, where $\rho_1^C$ denotes the frequency reuse factor used in the central region and $\rho_1^E$ denotes the frequency reuse factor used in the edge region, is applied in the first tier of macro BSs, shown in Fig. 5.1(a) in the hybrid-modeled HetNet, and shown in Fig. 5.1(b) in the PPP-modeled HetNet. The coverage area of each macro BSs is partitioned into two regions by the partitioning distance $D_1$. The central
region is the part of the macro BS coverage area with the distance not greater than $D_1$, and the frequency reuse factor $\rho_1^C$ is used in $W^C$, the frequency region allocated for macro BS users in the central region. As shown in Fig. 5.1, the universal frequency reuse, that is, $\rho_1^C = 1$, is commonly assumed for users in the central region of macro BSs. Meanwhile, the frequency reuse factor $\rho_1^E$ is applied in $W^E$, the frequency region for macro BS users in the edge region. For example, $\rho_1^E = 3$ is shown in Fig. 5.1. Specifically, for macro BSs with hexagonal tessellation in the hybrid HetNet, see Fig. 5.1(a), one of the three sub-bands $F_2$, $F_3$, and $F_4$ is allocated to the edge region of each macro BS, and the allocation follows the regular pattern, as stated in Chapter 4 (see the system assumption (A2) in Section 4.2 and [97]). For randomly positioned macro BSs in the PPP-modeled HetNet as shown in Fig. 5.1(b), the regular pattern of frequency allocation is impossible, instead, the sub-bands $F_2$, $F_3$, and $F_4$ are assumed randomly and independently allocated to each macro BS, as stated in Chapter 3 (see the system assumption (A2) in Section 3.2 and [17]). The frequency width ratio of $W^C$ to $W$ is assumed to be identical with $b$, the ratio of macro BS users in the central region to all macro BS users [70] [110] [131], derived later in Section 5.3.1. Note that this ratio $b$ is equivalent to and thus also denotes the probability of users in the central region when users associate to macro BSs. In this way, the macro BS users at both the central region and the edge region have the same amount of frequency resource available at the system level. In the second tier of randomly located small BSs, the integer frequency reuse factor $\rho_2$ with a random pattern [17] is assumed and independent of FFR in the first tier.

For simplicity, the path loss exponent in tier $j$, $\alpha_j = \alpha$, which means the path loss exponent is identical for both tiers, though the derivation in this chapter applies to the general case for differentiated path loss exponent $\alpha_j$ ($j = 1,2$).
5.3 Performance Analysis of HetNets with FFR

The probability $b$ of users staying in the central region with the partitioning distance $D_1$ when the users associate to macro BSs is derived first in this section, based on the biased association policy in (3.3). Then the interference model of each tier and its Laplace transform are discussed. After that, the coverage probability and spectral efficiency are achieved in both the hybrid-modeled and PPP-modeled HetNets under different radio propagation conditions when FFR is exploited in the tier of macro BSs.

5.3.1 Probability of users in central region when users associate to Macro BSs

The difference in the deployment of macro BSs results in the probability distribution of user distance to the closest macro BS, which is different between hybrid-modeled and PPP-modeled HetNets, and
correspondingly the probabilities of users in the central region when associating to macro BSs are discussed separately.

**A. Hexagonal Tessellated Macro BSs in Hybrid-modeled HetNets**

Based on the biased association policy (3.3), the considered user at distance $r_1$ associates to the first tier $l = 1$ when the distance $r_2$ from the user to the closest small BS is greater than $R_2$,

$$R_2 \triangleq \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} r_1$$

(5.1)

Following the same logic from (A2.1.2) to (A2.1.5), the probability density function of $r_1$ for users associating to the tier $l = 1$ is achieved, with $\alpha_l = \alpha$,

$$f_{r_1}(r_1, l = 1) = \frac{2r_1}{R_1^2} e^{-\lambda_{BS} \pi (\frac{B_2 P_2}{B_1 P_1})^{\frac{2}{\alpha}} R_1^2}$$

(5.2)

By integrating (5.2) over the whole range of $r_1$ in the hexagonal area, the probability that the considered user associates to the tier $l = 1$ is

$$\Pr\{l = 1\} = \int_{D_1}^{R_1} \frac{2r_1}{R_1^2} e^{-\lambda_{BS} \pi (\frac{B_2 P_2}{B_1 P_1})^{\frac{2}{\alpha}} R_1^2} dr_1 = \frac{1 - e^{-\lambda_{BS} \pi (\frac{B_2 P_2}{B_1 P_1})^{\frac{2}{\alpha}} D_1^2}}{\lambda_{BS} \pi (\frac{B_2 P_2}{B_1 P_1})^{\frac{2}{\alpha}} R_1^2}$$

(5.3)

By integrating (5.2) over the range of $r_1$ in the central region, divided by $\Pr\{l = 1\}$ in (5.3), the probability of users staying in the central region when they associate to the tier $l = 1$ is

$$b = \frac{1}{\Pr\{l = 1\}} \int_{0}^{D_1} \frac{2r_1}{R_1^2} e^{-\lambda_{BS} \pi (\frac{B_2 P_2}{B_1 P_1})^{\frac{2}{\alpha}} R_1^2} dr_1 = \frac{1 - e^{-\lambda_{BS} \pi (\frac{B_2 P_2}{B_1 P_1})^{\frac{2}{\alpha}} D_1^2}}{1 - e^{-\lambda_{BS} \pi (\frac{B_2 P_2}{B_1 P_1})^{\frac{2}{\alpha}} R_1^2}}$$

(5.4)

**B. Randomly located macro BSs in PPP-modeled HetNets**

Based on the biased association policy (3.3), the considered user at $r_1$ associates to the first tier $l = 1$ if the distance of the user to its closest small BS $r_2$ is greater than $R_2$, where
\[ R_2 \triangleq \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} r_1 \] (5.5)

Eqn. (5.5) means that if there exists any small BS inside a circular region of radius \( R_2 \) around the considered user, the user associates to the tier \( l = 2 \), otherwise the user associates to the first tier. Here let \( X \) denote the number of small BSs inside the circle, which is a Poisson random variable with parameter \( \lambda_2^{BS} \pi \left[ \left( \frac{B_2 P_2}{B_1 P_1} \right)^\frac{1}{\alpha} r_1 \right]^2 \), since small BS positions are modeled as a PPP with intensity of \( \lambda_2^{BS} \). Thus the probability of a user at \( r_1 \) associating to the first tier is the probability \( Pr\{X = 0\} \),

\[
Pr\{l = 1|r_1\} = Pr\{r_2 > R_2\} = Pr\{X = 0\} = e^{-\lambda_2^{BS} \pi \left( \frac{B_2 P_2}{B_1 P_1} \right)^\frac{2}{\alpha} r_1^2} \] (5.6)

Since users are uniformly distributed, the probability density function (PDF) of user distance \( r_1 \) to the nearest macro BS [17] [20] is

\[
f_{r_1}(r_1) = 2\pi \lambda_1^{BS} r_1 e^{-\pi \lambda_1^{BS} r_1^2}, 0 < r_1 \] (5.7)

Note that \( f_{r_1}(r_1) \) is the PDF for all users at \( r_1 \), including users associating to the first tier and users associating to the second tier. By multiplying (5.7) with (5.6), then the pdf of the user at the distance \( r_1 \) and in the tier \( l = 1 \) is

\[
f_{r_1}(r_1, l = 1) = 2\pi \lambda_1^{BS} r_1 e^{-\pi \lambda_1^{BS} r_1^2} \left( \frac{\lambda_2^{BS} (B_2 P_2)^\frac{2}{\alpha}}{\lambda_1^{BS} (B_1 P_1)^\frac{2}{\alpha}} \right) \] (5.8)

Integrating (5.8) over the whole range of \( r_1 \) gives the probability of users associating to the tier \( l = 1 \),

\[
Pr\{l = 1\} = \int_0^\infty 2\pi \lambda_1^{BS} r_1 e^{-\pi \lambda_1^{BS} r_1^2} \left( \frac{\lambda_2^{BS} (B_2 P_2)^\frac{2}{\alpha}}{\lambda_1^{BS} (B_1 P_1)^\frac{2}{\alpha}} \right) dr_1 = \frac{1}{1 + \lambda_2^{BS} (B_2 P_2)^\frac{2}{\alpha} (B_1 P_1)^\frac{2}{\alpha}} \] (5.9)
By integrating (5.8) over the range of $r_1$ in the central region, and then dividing by $Pr\{l = 1\}$, the probability of users staying in the central region when they associate to the tier $l = 1$ is

$$b = \int_{D_1}^{D_1} 2\pi \lambda_1^{BS} r_1 e^{-\pi r_1^2 \left( \frac{\lambda_1^{BS} + \lambda_2^{BS} (\frac{B_2 P_2}{B_1 P_1})^2}{\lambda_1^{BS} + \lambda_2^{BS} (\frac{B_2 P_2}{B_1 P_1})^2} \right)} \text{d}r_1 = 1 - e^{-\pi \left( \frac{\lambda_1^{BS} + \lambda_2^{BS} (\frac{B_2 P_2}{B_1 P_1})^2}{\lambda_1^{BS} + \lambda_2^{BS} (\frac{B_2 P_2}{B_1 P_1})^2} \right) D_1^2}$$

(5.10)

5.3.2 Tier interference models of HetNets with FFR

The tier interference models in HetNets with FFR are analyzed by formulating the intensity $\lambda_j$ of the effective interfering BSs from the tier $j$ and the radius $d_{ij}$ of interference free disc around the considered user associating to the tier $l$. After that, the statistics of interference from the tier $j$ is characterized by its Laplace transform. The previous results in Lemmas 3.1 and 3.2 from Chapter 3 are exploited for HetNets under different propagation situations. Recall that $L_{I_j}(s)$ in Lemmas 3.1 and 3.2 depend on $\lambda_j$, $d_{ij}$, and other parameters. In this chapter, this dependence on $\lambda_j$ and $d_{ij}$ is expressed explicitly and the notation $L_{I_j}(s \mid \lambda_j, d_{ij})$ is used to represent the results in Lemmas 3.1 and 3.2, instead of the notation $L_{I_j}(s)$ in Chapter 3, since the implementation of FFR on macro BSs makes the values of $\lambda_j$ and $d_{ij}$ varying under different cases.

A. Interference from tier of small BSs

For the considered user at $r_1$, there are only $\frac{1}{\rho_2}$-th of the small BSs sharing the radio resource with the considered user, since the independent random pattern of integer frequency reuse is applied to the second tier. Meanwhile, every co-channel small BS transmits with probability $\eta_2$. With joint consideration of frequency reuse factor $\rho_2$ and transmission probability $\eta_2$, the effective interfering BSs from the second tier $j = 2$ at any time instant form a PPP thinning with intensity $\lambda_2$.

$$\lambda_2 = \frac{\lambda_2^{BS} \eta_2}{\rho_2}$$

(5.11)
Instead, the radius \( d \) of interference free disc for the tier of small BSs \( (j = 2) \) depends on which tier \( i \) the considered user associates to. The interference free radius for users associating to the first tier \( (i = 1) \) is, as discussed in (5.1) and (5.5),
\[
d_{12} = \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} r_1
\]  
(5.12)
For users associating to the small BSs \( (i = 2) \), since the closest small BS is at \( r_2 \) away to the considered user, thus the distances of the other small BSs are no less than \( r_2 \). Thus,
\[
d_{22} = r_2
\]  
(5.13)
Consequently, the statistics of tier interference from small BSs is characterized by its Laplace transform as follows,
\[
\mathcal{L}_{I_2}(s) = \mathcal{L}_{I_j}(s|\lambda_2, d)
\]  
(5.14)
where \( d = d_{12} \) in (5.12) for users associating to the first tier and \( d = d_{22} \) in (5.13) for users associating to the second tier, and \( \lambda_2 \) is given in (5.11). On the right hand of (5.14), \( \mathcal{L}_{I_j}(s|\lambda, d) \) is given in Lemma 3.1 for Rician fading and in Lemma 3.2 for lognormal shadowed Rayleigh fading.

**B. Interference from tier of macro BSs**

For the interference from the first tier, the values of \( \lambda_1 \) and \( d_{11} \) depend on which tier the considered user associates to and the location of the user when FFR is applied. There are three cases discussed as follows.

1. **Users associating to macro BSs**

   When the considered user associating to the tier of macro BSs is located in the central region, the frequency reuse factor \( \rho_1 = \rho_1^C \) is applied in \( W^C \) thus \( \frac{1}{\rho_1^C} \)th of macro BSs share frequency with the
user. With further consideration of transmission probability $\eta_1$, the effective interfering BSs from the first tier $j = 1$ form a PPP thinning with the intensity parameter of $\lambda_1 = \lambda^C_1$, where

$$\lambda^C_1 = \lambda^{BS}_1 \eta_1 / \rho^C_1$$

(5.15)

If the user associating to the tier of macro BSs stays in the edge region, instead of the central region, the frequency reuse factor $\rho_1 = \rho^E_1$ is applied in $W^E$ (shown in Fig. 5.1). Correspondingly, the effective interfering BSs from the first tier $j = 1$ form a PPP thinning with the intensity parameter of $\lambda_1 = \lambda^E_1$, where

$$\lambda^E_1 = \lambda^{BS}_1 \eta_1 / \rho^E_1$$

(5.16)

Different from the intensity $\lambda_1$ which is independent of the deployment of macro BSs, the radius $d_{11}$ of interference free disc depends on the deployment of macro BSs. For hexagonal tessellated macro BSs in hybrid-modeled HetNets, the radius $d_{11}$ of interference free disc, as that in (4.1), is

$$d^{H,C}_{11} = \delta \times \left( 2R_M \sqrt{\rho^C_1} - r_1 \right)$$

(5.17)

for users in the central region, and

$$d^{H,E}_{11} = \delta \times \left( 2R_M \sqrt{\rho^E_1} - r_1 \right)$$

(5.18)

for users in the edge region. In (5.17) and (5.18), the superscript “H” denotes the hybrid model, the superscript “C” denotes the central region for FFR, and superscript “E” denotes the edge region for FFR.

Instead in PPP-modeled HetNets, each macro BS position is independent of the others. Under the constraint that the considered user associates to the nearest macro BS at $r_1$, all the interfering macro BSs must have the distance greater than $r_1$, which means the radius $d_{11}$ of the interference free disc is
\[ d_{11}^p = r_1 \] (5.19)

for all users associating to the tier of macro BSs, regardless of being located in the central region or the edge region.

After the modified BS intensity and the radius of the interference free disc are formulated, the interference from the macro BSs is characterized by its Laplace transform derived in Lemmas 3.1 and 3.2 as follows,

\[ \mathcal{L}_{I_1}(s) = \mathcal{L}_{I_f}(s|\lambda_1, d_{11}) \] (5.20)

where

a. \( \lambda_1 = \lambda_1^C \) for users in the central region and \( \lambda_1 = \lambda_1^E \) for users in the edge region;

b. In the hybrid-modeled HetNets, \( d_{11} = d_{11}^{H,C} \) in (5.17) for users in the central region and \( d_{11} = d_{11}^{H,E} \) in (5.18) for users in the edge region. Otherwise, \( d = d_{11}^p \) in (5.18) for PPP-modeled HetNets;

c. On the right hand of (5.20), \( \mathcal{L}_{I_f}(s|\lambda_1, d_{11}) \) is derived in Lemma 3.1 for Rician fading and in Lemma 3.2 for lognormal shadowed Rayleigh fading.

2. Users associating to small BSs

The frequency of the system is divided into two regions for FFR exploited macro BSs, \( W^C \) for users in the central region, and \( W^E \) for users in the edge region, as shown in Fig. 5.1(c). The second tier shares the frequency with the first tier and applies the integer frequency reuse factor \( \rho_2 \) which is independent of the frequency reuse in the first tier, as stated in the assumption A2 in the system model, which means that the frequency allocated to the considered user in the second tier is probably in any one of both frequency regions and the interference models of macro BSs in both frequency
regions need to be considered. The considered user associating to the tier of small BSs has the probability $b$ allocated in $W^C$ and the probability $(1 - b)$ in $W^E$. Correspondingly, the effective interfering BSs from the tier of macro BSs $j = 1$ form a PPP thinning with the intensity parameter of $\lambda_1 = \lambda_1^C$ in (5.15) for the frequency region $W^C$ and $\lambda_1 = \lambda_1^E$ in (5.16) for the frequency region $W^E$.

For both the frequency reuse factors $\rho_1 = \rho_1^C$ used for macro BSs in the frequency region $W^C$ and $\rho_1 = \rho_1^E$ for macro BSs in the frequency region $W^E$, the considered user has the probability $1/\rho_1$ of sharing the frequency with the closest macro BS and the probability $(\rho_1 - 1)/\rho_1$ of not sharing the frequency with the closest macro BS.

In hybrid-modeled HetNets, when the considered user allocates frequency in $W^C$ and shares the frequency with the closest macro BS which allocates it to users in the central region, the radius $d_{21}$ of interference free disc is the same as (5.17),

$$d_{21}^{H,C}(\rho_1) = \delta \times \left(2R_M\sqrt{\rho_1^C} - r_1\right)$$

(5.21)

When the considered user shares the frequency with the closest macro BS in $W^E$ (sharing the frequency with the closest macro BS which allocates it to users in the edge region), the radius $d_{21}$ of interference free disc is the same as (5.18),

$$d_{21}^{H,E}(\rho_1) = \delta \times \left(2R_M\sqrt{\rho_1^E} - r_1\right)$$

(5.22)

If the frequency not used by the closest macro BS, such as $F_3$ and $F_4$ are not used by the macro BS labeled with red-colored $A$ shown in Fig. 5.1(a), is allocated to the considered user at $r_1$ that associates to the second tier, the minimal distance from the considered user to co-channel macro
BSs is \( D = 2R_M - r_1 \). Correspondingly, the radius \( d_{21} \) of interference free disc under this situation is
\[
d_{21}^H O = \delta \times (2R_M - r_1)
\] (5.23)

Therefore, for users associating to the tier of small BSs, the Laplace transform of the interference from the tessellated macro BSs with FFR(\( \rho_1^C, \rho_1^E \)) in hybrid-modeled HetNets is,
\[
\mathcal{L}_{I_1}(s) = \begin{pmatrix}
\text{(a)}
E \{ e^{I_1} \} \\
\text{(b)}
b \cdot E \{ e^{I_1} \mid W^C \} + (1 - b) \cdot E \{ e^{I_1} \mid W^E \} \\
\text{(c)}
b \left[ \frac{1}{\rho_1^C} \cdot E \{ e^{I_{1,0} + I_{1,1}} \mid W^C \} + \frac{\rho_1^E - 1}{\rho_1^C} \cdot E \{ e^{I_{1,1}} \mid W^C \} \right] + (1 - b) \left[ \frac{1}{\rho_1^E} \cdot E \{ e^{I_{1,0} + I_{1,1}} \mid W^E \} \right. \\
\left. + \frac{\rho_1^E - 1}{\rho_1^E} \cdot E \{ e^{I_{1,1}} \mid W^E \} \right] \\
\text{(d)}
b \left( \frac{\mathcal{L}_{I_{1,0}}(s)}{\rho_1^C} \cdot \mathcal{L}_{I_{1,1}}(s \mid \lambda_1^C, d_{21}^{H,C}) + \frac{\rho_1^E - 1}{\rho_1^C} \cdot \mathcal{L}_{I_{1,1}}(s \mid \lambda_1^C, d_{21}^{H,0}) \right) + (1 - b) \left( \frac{\mathcal{L}_{I_{1,0}}(s)}{\rho_1^E} \cdot \mathcal{L}_{I_{1,1}}(s \mid \lambda_1^E, d_{21}^{H,E}) \right. \\
\left. + \frac{\rho_1^E - 1}{\rho_1^E} \cdot \mathcal{L}_{I_{1,1}}(s \mid \lambda_1^E, d_{21}^{H,0}) \right) 
\right)
\] (5.24)

where the definition of the Laplace transform is given in (a). (b) states the fact that the considered user associating to the tier of small BSs has the probability \( b \) of being allocated a frequency in the region \( W^C \) and the probability \( (1 - b) \) in the region \( W^E \). (c) considers whether or not the frequency allocated to the considered user is used by the closest macro BS, where \( I_{1,0} \) denotes the interference component from the closed macro BS and \( I_{1,1} \) expresses the interference component from the other macro BSs. The interference Laplace transform \( \mathcal{L}_{I_{1,0}} \) in (d) is derived in (A2.2.4), \( \mathcal{L}_{I_{1,1}}(s \mid \lambda_1^C, d_{21}^{H,C}), \mathcal{L}_{I_{1,1}}(s \mid \lambda_1^C, d_{21}^{H,0}), \mathcal{L}_{I_{1,1}}(s \mid \lambda_1^E, d_{21}^{H,E}) \) and \( \mathcal{L}_{I_{1,1}}(s \mid \lambda_1^E, d_{21}^{H,0}) \) are provided in Lemma 3.1 and 3.2.

As a common setting of FFR \( \rho_1^C = 1, d_{21}^{H,0} = d_{21}^{H}(\rho_1^C) \), and (5.24) simplifies to
\[ L_{I_1}(s) = b L_{I_{1,0}}(s) L_{I_{1,1}}(s|\lambda^C_{1}, d^{H,0}_{21}) + (1 - b) \left( \frac{L_{I_{1,0}}(s)}{\rho^E_{1}} L_{I_{1,1}}(s|\lambda^E_{1}, d^{H,E}_{21}) + \frac{\rho^E_{1} - 1}{\rho^E_{1}} L_{I_{1,1}}(s|\lambda^E_{1}, d^{H,0}_{21}) \right) \]  

(5.25)

In PPP-modeled HetNets, regardless of sharing the frequency with the closest macro BS or not, the radius of interference free disc (the interfering component from the closest macro BS is analyzed separately) is identical for the same reason of (5.19),

\[ d^E_{21} = r_{1} \]  

(5.26)

Thus, following the derivation approach similarly to (5.24), the Laplace transform of the interference from the randomly deployed macro BSs with \( FFR(\rho^C_{1}, \rho^E_{1}) \) in PPP-modeled HetNets is

\[ L_{I_1}(s) = b \left( \frac{L_{I_{1,0}}(s)}{\rho^C_{1}} + \frac{\rho^E_{1} - 1}{\rho^C_{1}} \right) L_{I_{1,1}}(s|\lambda^C_{1}, d^{P,E}_{21}) + (1 - b) \left( \frac{L_{I_{1,0}}(s)}{\rho^E_{1}} + \frac{\rho^E_{1} - 1}{\rho^E_{1}} \right) L_{I_{1,1}}(s|\lambda^E_{1}, d^{P,E}_{21}) \]  

(5.27)

where \( I_{1,0} \) denotes the interference component from the closest macro BS and \( I_{1,1} \) expresses the interference component from the other macro BSs, the interference Laplace transform \( L_{I_{1,0}} \) is derived in (A2.2.4), \( L_{I_{1,1}}(s|\lambda^C_{1}, d^{P,E}_{21}) \) and \( L_{I_{1,1}}(s|\lambda^E_{1}, d^{P,E}_{21}) \) are provided in Lemma 3.1 and 3.2 for different radio propagation situations.

As a common setting of FFR with \( \rho^C_{1} = 1 \), (5.27) is simplified to

\[ L_{I_1}(s) = b L_{I_{1,0}}(s) L_{I_{1,1}}(s|\lambda^C_{1}, d^{P,E}_{21}) + (1 - b) \left( \frac{L_{I_{1,0}}(s)}{\rho^E_{1}} + \frac{\rho^E_{1} - 1}{\rho^E_{1}} \right) L_{I_{1,1}}(s|\lambda^E_{1}, d^{P,E}_{21}) \]  

(5.28)

The previous discussions on the intensity of the tier interference and the radius of interference free disc around the considered user are summarized in Table 5.1 for hybrid-modeled HetNets and Table 5.2 for PPP-modeled HetNets, where \( \rho^C_{1} = 1 \) is assumed as the most common setting of FFR.
Table 5.1 Interference Model in Hybrid-modeled HetNets for the considered user at the distance $(r_1, r_2)$ to the nearest Macro BS and to the nearest small BS

<table>
<thead>
<tr>
<th>Radius $d_{ij}$ of interference-free disc and intensity of effective interfering BSs from tier $j$</th>
<th>Interference from tier $j = 1$</th>
<th>Interference from tier $j = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associating to Macrocells $l = 1$</td>
<td>$d_{11}^{H,c} = \delta \times (2R_M - r_1)$ and $\lambda_1^c = \lambda_1^{BS} \eta_1$ if $r_1 \leq D_1$; $\lambda_1^c = \lambda_1^{BS} \eta_1 / \rho_1^E$ if $r_1 &gt; D_1$;</td>
<td>$d_{12} = \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} r_1$</td>
</tr>
<tr>
<td>$r_2 &gt; R_2 = \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} \frac{\bar{a}<em>{21}}{r_1^{\bar{a}</em>{12}}} \frac{\bar{a}<em>{22}}{r_1^{\bar{a}</em>{12}}}$</td>
<td>$\lambda_2 = \lambda_2^{BS} \cdot \eta_2 / \rho_2$</td>
<td></td>
</tr>
<tr>
<td>$Pr(l = 1</td>
<td>r_1) = e^{-\lambda_1^{BS} \eta_1 / \rho_1^E}$</td>
<td>$d_{22} = r_2$</td>
</tr>
<tr>
<td>Associating to small cells $l = 2$</td>
<td>$1. Pr = b; d_{21}^{H,c} = \delta \times (2R_M - r_1)$, $\lambda_2^c = \lambda_2^{BS} \eta_1$ and co-channel closest macro BS; $2. Pr = (1 - b) / \rho_1^E; d_{21}^{HE} = \delta \times \left( 2R_M \sqrt{\rho_1^E} - r_1 \right)$, $\lambda_2^c = \lambda_2^{BS} \eta_1 / \rho_1^E$ and co-channel closest macro BS; $3. Pr = (1 - b) \left( \rho_2^E - 1 \right) / \rho_2^E; d_{21}^{H,t} = \delta \times (2R_M - r_1)$, $\lambda_2^t = \lambda_2^{BS} \eta_1 / \rho_1^E$</td>
<td>$\lambda_2 = \lambda_2^{BS} \cdot \eta_2 / \rho_2$</td>
</tr>
<tr>
<td>$r_2 \leq R_2 = \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} \frac{\bar{a}<em>{21}}{r_1^{\bar{a}</em>{12}}} \frac{\bar{a}<em>{22}}{r_1^{\bar{a}</em>{12}}}$</td>
<td>$d_{22} = r_2$</td>
<td></td>
</tr>
<tr>
<td>$Pr(l = 2</td>
<td>r_1) = 1 - Pr(l = 1</td>
<td>r_1)$</td>
</tr>
<tr>
<td>Distance distribution</td>
<td>$f_{r_1}(r_1) = 2r_1 / R_1^2$, $f_{r_1}(r_1,l) = f_{r_1}(r_1) Pr(l</td>
<td>r_1)$, $0 &lt; r_1 \leq R_1$ where $\pi R_1^2 = 2\sqrt{3}R_M^2$</td>
</tr>
</tbody>
</table>

Table 5.2 Interference Model in PPP-modeled HetNets for the considered user at the distance $(r_1, r_2)$ to the nearest Macro BS and to the nearest small BS

<table>
<thead>
<tr>
<th>Radius $d_{ij}$ of interference-free disc and intensity of effective interfering BSs from tier $j$</th>
<th>Interference from tier $j = 1$</th>
<th>Interference from tier $j = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associating to Macrocells $i = 1$</td>
<td>$d_{11} = r_1$; $\lambda_1 = \lambda_1^{BS} \cdot \eta_1 / \rho_1^E$ if $r_1 \leq D_1$ (5.1); $\lambda_1 = \lambda_1^{BS} \cdot \eta_1 / \rho_1^E$ if $r_1 &gt; D_1$ (5.2);</td>
<td>$d_{12} = \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} r_1$</td>
</tr>
<tr>
<td>$r_2 &gt; R_2 = \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} \frac{\bar{a}<em>{21}}{r_1^{\bar{a}</em>{12}}} \frac{\bar{a}<em>{22}}{r_1^{\bar{a}</em>{12}}}$</td>
<td>$\lambda_2 = \lambda_2^{BS} \cdot \eta_2 / \rho_2$</td>
<td></td>
</tr>
<tr>
<td>$Pr(l = 1</td>
<td>r_1) = e^{-\lambda_1^{BS} \eta_1 / \rho_1^E}$</td>
<td>$d_{22} = r_2$</td>
</tr>
<tr>
<td>Associating to small cells $i = 2$</td>
<td>$1. Pr = b; \lambda_1^c = \lambda_1^{BS} \eta_1$ and co-channel closest macro BS; $2. Pr = (1 - b) / \rho_1^E; \lambda_1^c = \lambda_1^{BS} \eta_1 / \rho_1^E$ and co-channel closest macro BS; $3. Pr = (1 - b) \left( \rho_2^E - 1 \right) / \rho_2^E; \lambda_1^t = \lambda_1^{BS} \eta_1 / \rho_1^E$</td>
<td>$\lambda_2 = \lambda_2^{BS} \cdot \eta_2 / \rho_2$</td>
</tr>
<tr>
<td>$r_2 \leq R_2 = \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{\alpha}} \frac{\bar{a}<em>{21}}{r_1^{\bar{a}</em>{12}}} \frac{\bar{a}<em>{22}}{r_1^{\bar{a}</em>{12}}}$</td>
<td>$d_{22} = r_2$</td>
<td></td>
</tr>
<tr>
<td>$Pr(l = 2</td>
<td>r_1) = 1 - Pr(l = 1</td>
<td>r_1)$</td>
</tr>
<tr>
<td>Distance distribution</td>
<td>$f_{r_1}(r_1) = 2\pi \lambda_1^{BS} r_1 e^{-\lambda_1^{BS} \pi r_1^2}$, $f_{r_1}(r_1,l) = f_{r_1}(r_1) Pr(l</td>
<td>r_1)$, $r_1 \in (0, \infty)$</td>
</tr>
</tbody>
</table>
5.3.3 Coverage probability and spectral efficiency

After formulating the probability of macro BS users in the central region and the interference Laplace transform from each tier, the analyses for coverage probability and spectral efficiency at \( r_1 \) are derived in this subsection. The following analytical results are expressed based on the propositions 4.1 to 4.3 in Chapter 4, for both hybrid-modeled and PPP-modeled HetNets with \( \lambda_j \) and \( d_{ij} \) listed in Tables 5.1 and 5.2, respectively. The analytical results in Chapter 4 depend on the intensity of the effective interfering BSs from the first tier \( \lambda_1 \) and other parameters, but the dependence is not denoted explicitly in the notations of Chapter 4, for example, \( p_{c,d}(T|r_1) \), since their values are not changed in the analyses in Chapter 4. However, the dependences on \( \lambda_1 \) will be expressed explicitly when these results are used in this chapter, since the implementation of FFR on macro BSs makes the values of \( \lambda_1 \) vary under different cases discussed in 5.3.1. For example, the notation \( p_{c,d}(T|r_1, \lambda_1) \), instead of \( p_{c,d}(T|r_1) \) used in Chapter 4, will be adopted in this chapter.

As discussed in 5.3.2, FFR implementation on the tier of macro BSs results in the dependence of the interference model on which tier the considered user associates to and where the user is currently located at. Correspondingly, the coverage probability at \( r_1 \) is presented for users in these different cases, respectively. After that, the coverage probability of each tier and in the whole network are provided, followed by spectral efficiency.

A. Coverage probability for users at \( r_1 \) in central region when associating to macro BSs

For the case of the considered user at \( r_1 \leq D_1 \) associating to the tier of macro BSs, the interference from the tier of small BSs is the PPP thinning with the intensity parameter of \( \lambda_2 \) in (5.11) and the radius of the interference free disc \( d_{12} \) in (5.12) determined by the biased association policy. Meantime, the interference from the first tier is seen as the PPP thinning with the intensity parameter
of $\lambda_1 = \lambda_1^C$ in (5.15) and the radius of the interference free disc $d_{11}$, where $d_{11} = d_{11}^{H,C}$ in (5.17) for hybrid-modeled HetNets and $d_{11} = d_{11}^{P}$ in (5.19) for PPP-modeled HetNets. Thus the coverage probability for users at $r_1 \leq D_1, p_{c,t=1}(T|r_1 \leq D_1, \lambda_1)$, is derived as presented in Propositions 4.1 to 4.3 for different radio propagation situations, with the parameters setting of the aforementioned $\lambda_j$ and $d_{lj}$ which are changed due to FFR implementation in macro BSs.

B. Coverage probability for users at $r_1$ in edge region when associating to macro BSs

For the considered user in the edge region ($r_1 > D_1$) associating to the tier of macro BSs, the interference from the tier of small BSs is still seen as the PPP thinning with the intensity parameter of $\lambda_2$ in (5.11) and the radius of the interference free disc $d_{12}$ in (5.12), while the interference from the first tier is reduced, since the edge frequency reuse factor $\rho_1^E$ is used instead of $\rho_1^C$, resulting in the intensity parameter of $\lambda_1 = \lambda_1^E$ (5.16). The radius of the interference free disc for this case is $d_{11} = d_{11}^{H,E}$ in (5.18) for hybrid-modeled HetNets and $d_{11} = d_{11}^{P}$ in (5.19) for PPP-modeled HetNets. Thus the coverage probability for users at $r_1 > D_1, p_{c,t=1}(T|r_1 > D_1, \lambda_1)$, is derived as presented in Propositions 4.1 to 4.3 for different radio propagation situations with the parameters setting of the aforementioned $\lambda_j$ and $d_{lj}$ which are changed due to FFR implementation on macro BSs.

C. Coverage probability for users at $r_1$ associating to small BSs

For the considered user at $r_1$ associating to the tier of small BSs, the interference from the tier of small BSs is seen as the PPP thinning with the intensity parameter of $\lambda_2$ (5.11) and the radius of the interference free disc $d_{22}$ in (5.13). The interference from the first tier is the probability weighted combination of PPPs and the Laplace transform is derived in (5.24) for hybrid-modeled HetNets
and in (5.27) for PPP-modeled HetNets. Thus the coverage probability for users at \( r_1 \) associating to the tier of small BSs, \( p_{c, l=2}(T | r_1, \lambda_1) \), is derived as presented in Propositions 4.1 to 4.3 for different radio propagation situations, with the modified Laplace transform of the interference from the tier of macro BSs due to FFR implementation on the first tier.

**D. Tier coverage probability and network coverage probability**

Integrating the coverage probability of each tier at \( r_1 \) over the corresponding pdf of \( r_1 \) for each tier \( f_{r_1}(r_1 | l), l = 1, 2 \) and \( f_{r_1}(r_1) \), shown in Tables 5.1 and 5.2, gives the tier coverage probabilities for the tier \( l = 1 \) in (5.29), \( l = 2 \) in (5.30), and the network coverage probability in (5.31), respectively.

\[
p_{c, l=1}(T) = \int_0^{D_1} p_{c, l=1}(T | r_1, \lambda_1^C) f_{r_1}(r_1 | l = 1) dr_1 + \int_{r_1 > D_1} p_{c, l=1}(T | r_1, \lambda_1^E) f_{r_1}(r_1 | l = 1) dr_1 (5.29)
\]

\[
p_{c, l=2}(T) = \int_0^{R_1} p_{c, l=2}(T | r_1, \lambda_1^E) f_{r_1}(r_1 | l = 2) dr_1 (5.30)
\]

\[
p_c(T) = \int_0^{R_1} p_c(T | r_1) f_{r_1}(r_1) dr_1 = p_{c, l=1}(T) Pr\{l = 1\} + p_{c, l=2}(T) Pr\{l = 2\} (5.31)
\]

where \( p_{c, l=1}(T | r_1 \leq D_1, \lambda_1^C) \), \( p_{c, l=1}(T | r_1 > D_1, \lambda_1^E) \), and \( p_{c, l=2}(T | r_1, \lambda_1^E) \) are given in Parts A to C of this subsection, respectively, and \( f_{r_1}(r_1 | l = 1) \), \( f_{r_1}(r_1 | l = 2) \), \( Pr\{l = 1\} \), and \( Pr\{l = 2\} \) can be derived from the analytical results shown in Tables 5.1 and 5.2 for the hybrid-modeled and PPP-modeled HetNets, respectively.

**E. Spectral efficiency**

The spectral efficiency has been derived in Proposition 4.4 provided the corresponding coverage probability is known, and they are presented as follows for completeness.

\[
\tau_1(r_1) = \log_2(e) \int_{T \geq 0} \frac{1}{T + G} p_{c, l}(T | r_1, \lambda_1) dT (5.32)
\]

\[
\tau_l = \int_{T \geq 0} \frac{1}{T + G} p_{c, l}(T) dT (5.33)
\]
\[ \tau = \int_{T>0} \frac{1}{T+G} p_c(T) dT \]  

(5.34)

where \( p_c(T|\lambda_1) \) is given in Parts A to C, \( p_{c,l}(T) \) in (5.29) for \( l = 1 \) and in (5.30) for \( l = 2 \), \( p_c(T) \) in (5.31). Notice that in (5.32) \( p_{c,l=1}(T|\lambda_1) = p_{c,l=1}(T|\lambda_1^2) \) when \( r_1 \leq D_1 \); \( p_{c,l=1}(T|\lambda_1^2) \) otherwise.

Table 5.3 Assumed System Parameter Values [21] [28]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Assumed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_M )</td>
<td>Apothem of hexagon</td>
<td>1500m ((\lambda_1 = 0.128/\text{km}^2))</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>Intensity of small BSs</td>
<td>0.128/\text{km}^2</td>
</tr>
<tr>
<td>( P = [P_1, P_2] )</td>
<td>Transmit power of tiers 1 and 2</td>
<td>[46, 26]dBm</td>
</tr>
<tr>
<td>( \alpha = [\alpha_1, \alpha_2] )</td>
<td>Path loss exponent of tiers 1 and 2</td>
<td>[3.76, 3.76]</td>
</tr>
<tr>
<td>( B = [B_1, B_2] )</td>
<td>Association bias of tiers 1 and 2</td>
<td>[1, 2]</td>
</tr>
<tr>
<td>( \sigma = [\sigma_1, \sigma_2] )</td>
<td>Standard deviation of shadowing in tiers 1 and 2</td>
<td>[0, 0]</td>
</tr>
<tr>
<td>( K = [K_1, K_2] )</td>
<td>Rician factor for the desired signal in tiers 1 and 2</td>
<td>[0, 0]</td>
</tr>
<tr>
<td>( K_i = \begin{bmatrix} K_{11} &amp; K_{12} \ K_{21} &amp; K_{22} \end{bmatrix} )</td>
<td>Rician factor ( k_{ij} ) for the interfering signals from tier ( j ) to tier ( l )</td>
<td>( \begin{bmatrix} 0 &amp; 0 \ 0 &amp; 0 \end{bmatrix} )</td>
</tr>
<tr>
<td>( \eta = [\eta_1, \eta_2] )</td>
<td>Transmission probability in tiers 1 and 2</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>( G )</td>
<td>SIR gap from Shannon capacity</td>
<td>3dB</td>
</tr>
<tr>
<td>( D_1 )</td>
<td>FFR partitioning distance</td>
<td>1000m ((= \frac{2}{3}\lambda_1))</td>
</tr>
<tr>
<td>( \rho_1^c )</td>
<td>Central region frequency reuse factor for macro BSs</td>
<td>1</td>
</tr>
<tr>
<td>( \rho_1^e )</td>
<td>Edge region frequency reuse factor for macro BSs</td>
<td>3</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>Frequency reuse factor in the tier of small BSs</td>
<td>3</td>
</tr>
</tbody>
</table>

5.4 Numerical Results and Discussion

This section presents numerical results, validated by simulation results, to demonstrate the usefulness of the derived analytical results and to comprehensively investigate the performance trend of FFR in two types of 2-tier heterogeneous cellular network with increasing small BS to
macro BS intensity ratio, in addition to the impact of Rician fading, shadowing, and transmission probability. Except noted otherwise, the assumed values of parameters used in the numerical evaluation in this chapter are listed in Table 5.3, where the channel parameters are mainly borrowed from [21] and [28], and the subscripts “1” and “2” denote the tier of macro BSs and the tier of small BSs, respectively.

FFR(1,3), as the optimal setting of fractional frequency reuse [105], is widely adopted in the literature [70] [85] and is also assumed in this section. Performance results of HetNets with FFR are compared to that with the universal frequency reuse (instead of FFR) used in the tier of macro BSs, since the universal frequency reuse provides the best system spectral efficiency among integer frequency reuse schemes [105].

5.4.1 Validating proposed analysis through simulation

The details of the simulation follows the depiction given in Section 4.4.1, except the steps (B1), (B2), and (B4) with some differences outlined as follows.

(B1) For macro BSs in the hybrid-modeled HetNet, each center of hexagonal area has one macro BS in hexagonal tessellation with apothem \( R_M \) over a circular service area of radius \( R_S = 50 \text{km} \); for small BSs in both types of HetNets and macro BSs in the PPP-modeled HetNet, the number of BSs follows Poisson distribution with the corresponding parameter \( \pi R_S^2 \lambda_i^{BS} \) and each BS is uniformly and independently dropped.

(B2) For the hybrid-modeled HetNet, the considered user uniformly locates in the central hexagonal area. For the PPP-modeled HetNet, the considered user uniformly locates in the central area with radius 3km. After that, the user associates to the tier based on the biased association policy in (3.3).
(B4) The macro BSs are partitioned into frequency groups $\rho_1^C$ and $\rho_1^E$ in the frequency region of width $W^C$ and $W^E$, respectively, following the regular pattern of frequency reuse in the hybrid-modeled HetNet while following a random pattern of frequency reuse in the PPP-modeled HetNet. Small BSs are randomly allocated into frequency groups $\rho_2$. After that, transmission slots are allocated to BSs in each tier with the corresponding tier transmission probability $\eta = [\eta_1, \eta_2]$. The simulation results are represented by marks and the analytical results by curves in Figs. 5.2 and 5.3 for the hybrid-modeled HetNet with FFR, and in Fig 5.4 for the PPP-modeled HetNet with FFR. All the results of the proposed approach in Figs 5.2 to 5.4 are almost overlapped with corresponding simulation results, thus confirming the accuracy of the analytical results. The simulation results are also presented by marks in the following figures, for further validation of the proposed approach under different settings and scenarios.

5.4.2 Trade-off between coverage probability and spectral efficiency

First, the performance of the hybrid-modeled HetNet with FFR is investigated. FFR provides significant improvement on the coverage probability in the edge region of macro BSs compared to the universal frequency reuse for macro BSs, since the higher reuse factor in the edge region increases the minimal distance to co-channel macro BSs and simultaneously lowers the intensity of co-channel macro BSs, as shown in Fig. 5.1(a), resulting in extensive interference reduction from neighboring macro BSs. As shown in Fig. 5.2(a), the coverage probability of macro BSs at the distance $r_1 = R_M$ substantially increases from 60% to 95% for SIR threshold $T = -4$dB when FFR(1,3) instead of $\rho_1 = 1$ is applied to macro BSs. For all users associating to macro BSs, the tier coverage probability is presented in Fig. 5.2(b), whose improvement is also considerable, from 83%
to 97% of coverage probability for the SIR threshold $T = -4 dB$ when FFR(1,3) is used instead of the universal frequency reuse for the tier of macro BSs.

Correspondingly to the coverage enhancement for users associating to the tier of macro BS in the edge region, the spectral efficiency is also significantly improved in the edge region ($r_1 \in [D_1, R_1]$). For example, as shown in Fig. 5.2(c), the spectral efficiency of macro BSs at $r_1 = 0.7 R_M$ increases by 253% from 1.51bits/s/Hz to 3.82bits/s/Hz when FFR(1,3) instead of the universal frequency reuse is exploited at the macro BSs. Recall that the spectral efficiency is the ratio of throughput to the frequency bandwidth allocated to the radio link. If the considered user is in the edge region of macro BS $A$ in Fig. 5.1, and the user associates to macro BS $A$, then macro BS $A$ will allocate $F_2$ for the downlink transmission, the spectral efficiency $\tau$, as defined in (3.4), is the ratio of the available information rate to the bandwidth of $F_2$. However, from the perspective of the system, though there is only one third of frequency resource in $W^E$ used for the downlink in the edge region since $\rho_1^E = 3$, the serving macro BS cannot exploit the remaining frequency resource in $W^E$ anymore, such as $F_3$ and $F_4$ which are reserved for users in the edge region of neighboring macro BSs. System spectral efficiency $\tau_s$, defined as the ratio of the available information rate to the bandwidth of $W^E$, including $F_2$ used for the downlink, $F_3$ and $F_4$ not used in order to reduce the interference among neighboring macro BSs, is the more appropriate measure at the system level. Mathematically, $\tau_s = \tau / \rho_1^E$ for macro BS users in the cell edge region. In this way, FFR(1,3) leads to a 16% reduction in the system spectral efficiency compared to the universal frequency reuse (1.51bits/s/Hz vs. 1.27bits/s/Hz), as shown in Fig. 5.2(d), instead of the aforementioned increase of the spectral efficiency by 253%.

135
Fig. 5.2 Performance evaluation of the tier of Macro BSs in the Hybrid-modeled HetNet with FFR(1,3)
As this example shows, though FFR provides significant improvement on coverage probability and thus the link spectral efficiency for users in the edge region compared to the universal frequency reuse, this improvement comes with the cost of fewer available frequency resource and sometimes lower spectral efficiency at the system level. Also notice that in Figs. 5.2(c) and 5.2(d), if \( r_1 = R_M \), the increase of spectral efficiency is up to 388\% from 0.56bits/s/Hz to 2.17bits/s/Hz, and the system spectral efficiency is improved by 129\% instead. In the previous research [131], it is said that the coverage advantage of frequency reuse comes with the cost of lower spectral efficiency. However, this observation of location-dependent metrics shows that, FFR in the hybrid-modeled HetNet has potential to improve the system spectral efficiency in addition to enhancing the coverage probability with an appropriate partitioning. The further investigation on this is left to Section 5.5.

It is interesting to know that the coverage probability and then the spectral efficiency of small BSs is also improved when FFR(1,3) is used in the tier of macro BSs instead of the universal frequency reuse, as shown in Figs. 5.3(a) and 5.3(b), respectively. The reason is, there is the probability \( 1 - b = 60\% \) that users associating to macro BSs stay in the edge region, where only one third of frequency resource in \( W^E \) is used for each macro BS due to \( \rho_1^E = 3 \). The probability of each macro BS interfering with the transmission in the second tier is only \( 1/\rho_1^E = 1/3 \) if the transmission of the second tier is in the frequency region \( W^E \). Instead, when the universal frequency reuse scheme is exploited by macro BSs, all the macro BSs interfere with the transmission in the second tier, which gives lower coverage probability and then lower spectral efficiency.
Move to the PPP-modeled HetNet with FFR as shown in Fig. 5.4 for its performance. Briefly, the coverage probability and then the spectral efficiency for both tiers are improved when FFR is applied for macro BSs, similarly to that in the hybrid-modeled HetNet. For SIR threshold $T = -4$ dB, the first tier coverage probability is improved from 73% to 87%, shown in Fig. 5.4(a), and the second tier coverage probability increases from 59% to 72%, shown in Fig. 5.4(b), when FFR(1,3) instead of the universal frequency reuse $\rho_1 = 1$ is applied to the tier of macro BSs. However, with the higher reuse factor for macro BS users in the edge region, the system spectral efficiency of the PPP-modeled HetNet is always lower when FFR(1,3) instead of $\rho_1 = 1$ is applied to the tier of macro BSs, as shown in Fig. 5.4 (d).

Fig. 5.3 Performance evaluation of the tier of small BSs in the Hybrid-modeled HetNet with FFR(1,3)
Fig. 5.4 Performance in the PPP-modeled HetNet with FFR(1,3)
This is different from that in the hybrid-modeled HetNets. The reason for this difference is provided in Section 4.5.1 and not repeated here, for conciseness. Thus, for PPP-modeled HetNets with FFR, the coverage enhancement comes always with the cost of lower system spectral efficiency. With the small partitioning distance $D_1$, the tier and network coverage probabilities of PPP-modeled HetNets are further enhanced, however, with the cost of further reduced system spectral efficiency.

5.5 Optimal Partitioning of FFR in terms of Spectral Efficiency for Hybrid-modeled HetNets

This chapter focuses on the investigation of FFR benefits and the optimal partitioning between the central region and the edge region in terms of system spectral efficiency in hybrid-modeled HetNets. Partitioning is said to be optimal if a partitioning distance $D_1$ (dividing the cell into two regions) maximizes the system spectral efficiency of macro BSs where FFR applies:

$$
\max_{D_1} \tau_1(D_1) = \int_0^{D_1} \tau_{s,l=1}(r_1|\rho_1^c) f_{r_1}(r_1|l = 1) dr_1 + \int_{r_1 > D_1} \tau_{s,l=1}(r|\rho_1^E) f_{r_1}(r_1|l = 1) dr_1 \quad (5.35)
$$

An example to demonstrate how to determine the optimal partitioning distance $D_1$ is shown in Fig. 5.2 for hybrid-modeled HetNets with the system parameters setting listed in Table 5.3 and the Rayleigh fading situation. As shown in Fig. 5.2(a) the dotted curves represent the system spectral efficiency for the universal frequency reuse, $\tau_{s,l=1}(r_1|\rho_1 = 1)$, which provides a higher system spectral efficiency for users close to the serving macro BS, because the users are sufficiently far away from the nearest interfering macro BSs. In this region, the higher frequency reuse factor such as $\rho_1 = 3$ gives lower frequency reuse due to the available frequency reduction. As the distance of users to their serving macro BSs becomes greater than the optimal partitioning distance $D_1^* = 0.81R_M$, $\tau(r|\rho_1 = 3)$ provides better system spectral efficiency instead, since the increase of
frequency reuse factor from $\rho_1 = 1$ to 3 significantly reduces the interference from neighboring macro BSs whose positive impact outperforms the negative impact of available frequency reduction.

Based on this location-dependent spectral efficiency analysis, one conclusion arrives that FFR(1,3) applied to macro BSs in hybrid-modeled HetNets with the assumed system parameters setting in Table 5.3 has the potential to achieve the higher system spectral efficiency compared to the universal frequency reuse, or other integer frequency reuse schemes. As stated in [105], the universal frequency reuse provides the best spectral efficiency among integer frequency reuse scheme. However, compared to the universal frequency reuse ($\rho_1 = 1$) achieving the system spectral efficiencies (0.69bits/s/Hz for macro BSs edge region and 1.42bits/s/Hz for small BSs), FFR with the optimal partitioning distance $D_1^* = 0.81R_M$ provides significant improvement, increases system
spectral efficiency by 18% for users in the edge region associating to macro BSs (0.82 bits/s/Hz) and 31% for users associating to small BSs (1.86 bits/s/Hz). Also shown in Fig. 5.2(a), when the FFR partitioning distance $D_1$ decreases from $D_1^*$, the cross-point of curves for reuse factors 1 and 3, $\tau_1(D_1)$ in (5.35) drops. In the reverse direction with the increase of $D_1$ from $D_1^*$, $\tau_1(D_1)$ still drops. Based on this observation, $\tau_1(D_1)$ in (5.35) is a concave function of $D_1$ and the optimal partitioning distance $D_1^*$ can be solved by the bisection search algorithm [157].

5.5.1 Impact of transmission probability on FFR

![Graphs illustrating FFR optimal partitioning and system spectral efficiency improvement](image)

(a) FFR optimal partitioning in consideration of transmission probability  
(b) System spectral efficiency improvement for macro BSs in the edge region

Fig. 5.6 Impact of transmission probability on system spectral efficiency improvement of FFR(1,3) with the optimal partitioning compared to the universal frequency reuse

As shown in Fig. 5.6(a), when the transmission probability $\eta_1$ assumes low to medium value (e.g. $\eta_1 = 0.6$), the total interference power becomes low, and the improvement of the system spectral efficiency for $\rho_1^E = 1$ becomes greater than that for $\rho_1^E = 3$ near the optimal point $D_1^* = 0.81R_M$
(for $\eta_1 = 1$), thus pushing the optimal partitioning point away from the BS. At the same time, the improvement of the system spectral efficiency in the edge area decreases because the region near the optimal distance $D_1^*$ has similar system spectral efficiencies and accounts for a greater proportion when $D_1^*$ increases. When the transmission probability increases from $\eta_1 = 0.5$ to $\eta_1 = 1.0$, the improvement of the system spectral efficiency in the edge region increases from less than 2% to 18%, as shown in Fig. 5.6(b). When the transmission probability is lower than $\eta_1 = 0.5$, FFR always underperforms the universal frequency reuse at any value of $r_1$ in terms of system spectral efficiency instead.

Fig. 5.7 Impact of shadowing on system spectral efficiency improvement of FFR(1,3) with the optimal partitioning compared to the universal frequency reuse
5.5.2 Impact of shadowing on FFR

For the hybrid-modeled HetNet under the lognormal shadowed Rayleigh fading, the shadowing has a significantly more negative impact on system spectral efficiency for the larger values of frequency reuse factor. The system spectral efficiency for $\rho_1^F = 3$ goes lower than that for $\rho_1^C = 1$ near the optimal point $D_1^* = 0.81R_M$ (for $\eta_1 = 1$), shown in Fig. 5.7(a), thus pushing the optimal partitioning point away from the BS and decreasing the improvement of the system spectral efficiency in the edge area, a similar situation with the decrease of transmission probability. As shown in Fig. 5.7(b), the benefit for FFR(1,3) decreases with the increase of shadowing standard deviation. For example, FFR(1,3) achieves 18% improvement in system spectral efficiency in the edge region for Rayleigh fading without shadowing ($\sigma_1 = 0$dB). This improvement is considerably higher compared to about 5% improvement for $\sigma_1 = 8$dB. When the shadowing standard deviation increases to 12dB or higher, FFR(1,3) is no longer beneficial with respect to system spectral efficiency compared to the universal frequency reuse, since at any point $r_1 \in (0, R_1]$, the system spectral efficiency for $\rho_1^F = 3$ is always lower than that for $\rho_1^C = 1$.

5.5.3 Impact of Rician fading on FFR

Different from shadowing which has negative impact on system spectral efficiency, LOS existence improves the system spectral efficiency for both reuse factors $\rho_1^C = 1$ and $\rho_1^F = 3$, shown in Fig. 5.8(a), thus keeping the optimal partitioning point relatively fixed, so is the benefit for FFR(1,3) with the increasing Rician factor from $K_1 = 0$ to $K_1 = 10$, always around 17% to 19% improvement in system spectral efficiency in the edge region shown in Fig. 5.8(b). This observation shows that the benefit of FFR in the hybrid-modeled HetNet is almost not related to whether or not LOS exists.
Fig. 5.8 Impact of Rician fading on system spectral efficiency improvement of FFR(1,3) with the optimal partitioning compared to the universal frequency reuse

5.5.4 Impact of SIR gap on FFR

Shown in Fig. 5.9(a), the increase of SIR gap $G$ draws the optimal partitioning point to the serving BS and provides a high benefit to system spectral efficiency, in contrast to the impact of shadowing in Section 5.4.2. As shown in Fig. 5.9(b), at $\sigma_1 = 8$dB and when the SIR gap from Shannon capacity $G_{dB}$ increases from 1dB to 6dB, the improvement of FFR increases from 2% to 11%. Instead, in Rayleigh fading without shadowing environment ($\sigma_1 = 0$dB), the improvement of the system spectral efficiency in the edge region increases basically linearly from 8% to 25%, over the same range of $G_{dB}$ considered. The reason is, with the increase of SIR gap $G$, the decrease of the system spectral efficiency for $\rho_1^C = 1$ becomes greater than that for $\rho_1^E = 3$, drawing the optimal partitioning point to the BS and thus FFR achieves increasing improvement in the system spectral
efficiency over the universal frequency reuse in the edge region of macro BSs. Recall that the reason for the difference of system spectral efficiency improvement between $\sigma_1 = 8dB$ and $\sigma_1 = 0dB$ has been discussed in Section 5.5.2.

![Graph showing system spectral efficiency improvement](image)

**Fig. 5.9** Impact of SIR gap on system spectral efficiency improvement of FFR(1,3) with the optimal partitioning compared to the universal frequency reuse

### 5.5.5 Impact of densifying small BSs on FFR

Though the higher frequency reuse factor in the edge region $\rho_1^E = 3$ significantly reduces the interference from the neighboring macro BSs compared to $\rho_1^C = 1$, however, the interference from the small BSs is untouched. With the increasing intensity of small BSs, the interference from the tier of small BSs becomes more and more dominant and the SIR improvement due to reduced interference from macro BSs by FFR does not compensate the loss of less available frequency resource for the larger frequency reuse factor. When the intensity ratio between the tier of small BSs
and the tier of macro BSs increases from $\lambda_2/\lambda_1 = 1$ to 8, the system spectral efficiency for $\rho_1^E = 3$ goes down faster than that for $\rho_1^C = 1$, shown in Fig. 5.10(a), thus the increasing ratio $\lambda_2/\lambda_1$ pushes the optimal partitioning point away from the BS and decreases the improvement of the system spectral efficiency in the edge area. As shown in Fig. 5.10(b), FFR(1,3) achieves about 20% improvement in system spectral efficiency in the edge region for homogeneous cellular network ($\lambda_2 = 0, \lambda_1 \neq 0$). This improvement is considerably higher compared to about 5% improvement for $\lambda_2/\lambda_1 = 8$. When the small BS to macro BS intensity ratio increases to 12 or higher, FFR(1,3) is no longer beneficial with respect to system spectral efficiency compared to the universal frequency reuse.

![Graphs showing the impact of increasing intensity of small BSs on system spectral efficiency improvement](image)

(a) FFR optimal partitioning in consideration of increasing intensity of small BSs  (b) System spectral efficiency improvement for macro BSs in the edge region

Fig. 5.10 Impact of increasing intensity of small BSs on system spectral efficiency improvement of FFR(1,3) with the optimal partitioning compared to the universal frequency reuse
5.6 Summary

Performance analyses are developed for both hybrid-modeled andPPP-modeled HetNets with FFR used in the tier of macro BSs, and the proposed analyses successfully characterize the coverage probability and the spectral efficiency under different radio propagation situations, including the Rayleigh fading situation, the Rician/Rayleigh fading situation, and the lognormal shadowed Rayleigh fading situation. After that, the improvement of system spectral efficiency and the optimal partitioning of FFR in terms of system spectral efficiency are evaluated comprehensively for hybrid-modeled HetNets. Based on these analyses and investigation, there are some key conclusions drawn. First, the developed analyses of location-dependent coverage probability and spectral efficiency present more details for interference management in HetNets, and, based on these, FFR in two-tier HetNets is evaluated which significantly enhances the coverage probability of macro BSs in the edge region, in addition to macro BS interference reduction for users in the second tier. Second, though the coverage enhancement of PPP-modeled HetNets with FFR comes with the cost of lower system spectral efficiency, hybrid-modeled HetNets with FFR have potential to provide the improvement of system spectral efficiency in addition to the coverage enhancement. This different conclusion between two types of HetNets comes from the difference in the deployment of macro BSs in the HetNet models considered: hexagonal tessellated in hybrid-modeled HetNets while randomly located in PPP-modeled HetNets. Third, the system spectral efficiency advantage of FFR in hybrid-modeled HetNets depends on the transmission probability, shadowing standard deviation, and the intensity of small BSs. The comprehensive investigation shows that, in addition to coverage enhancement, FFR is also beneficial to system spectral efficiency in hybrid-modeled HetNets under high transmission probability, low shadowing standard deviation, and lightly deployed small BSs.
6.1 Introduction

By manipulating signal transmission in the spatial domain, multiple antenna technologies have been used to mitigate the impact of interference, in addition to the capabilities of bringing multiplexing and diversity gains to boost the coverage probability and throughput [19] [158]. Multiple antenna technologies play an essential role in meeting the requirements of current and future radio access networks in the industry and have gained overwhelming interest in academia during the last decade [112]. The performance benefits due to array gain, diversity gain, and multiplexing gain have been investigated and analyzed considered with only a single BS [19] or in a multiuser scenario [159]. When HetNets with multiple antenna technologies are involved, most of the studies resort to Monte Carlo simulation [117] or field measurement [160]. The coverage probability of multiple antenna transmission in ad hoc networks is investigated in [161] where users are located at a fixed distance without consideration of the association policy. Analysis of the benefits of multiple antennas configured at the BSs in HetNets is desired and forms the goal of this chapter, with consideration of the impact of the association policy and intra-tier and inter-tier interference mitigation due to multiple antenna technologies.

Multiple antenna technologies have included a variety of techniques and the focus of this chapter is restricted to two flexible and effective precoding techniques that exhibit low complexity. These two precoding techniques are each used under situations representing opposite extremes for fading correlation among multiple antennas at each BS. The first one is called classical beamforming (BF) [97] [30] [162] which is applied under the fully correlated fading condition: multiple antennas of each BS are closely spaced and thus the fading is fully correlated among multiple antennas of the
BS [163]. As a versatile approach to spatial filtering without request for channel state information knowledge feedback, BF increases the signal power at the intended user and reduce the interference to non-intended users [19]. Notice that the direction of arrival can be estimated at the BS end in the uplink [164]. Recently, BF with massive antennas together have been seen as a promising enabling technology to cope with the unfavorable path loss at the millimeter wave frequency bands for next generation of radio access networks [121] [122] [165]. In this chapter, the impact of BF on the intra-tier and inter-tier interference and thus on the performance metrics of the coverage probability and spectral efficiency will be investigated for HetNets under the fully correlated fading condition.

The second multiple antenna technique is transmit diversity (TD) [116] [118] under the independent fading situation: spacing the antennas relatively far apart enables. TD is especially effective at mitigating the desired power variation due to multipath fading. At the same time, transmit diversity with multiple antennas results in multiple sources of interference to other users, the interference environment will be different from that of a HetNet with one transmit antenna at the transmitter. Though various TD methods such as have been proposed, some of which have been incorporated into standards [120] [166] [167], this thesis does not intend to compare the performance of different transmit diversity methods, but rather to evaluate the performance impact of the TD-introduced differences on the desired signal power and the interference statistics in HetNets. Notice that the analyses developed in this chapter can be applied to spatial time block coding techniques without channel knowledge at the transmitter, such as Alamouti code for 2 transmit antenna configuration [120], and space-frequency block codes (SFBCs) in long term evolution (LTE) for 2 or 4 antenna configurations [167].
In addition to classical beamforming providing array gain and transmit diversity offering diversity gain, there are some other multiple antenna techniques developed and intensively discussed in the literature with the hope of achieving spatial multiplexing gains, primarily through multi-user multiple-input and multiple-output (MU-MIMO [30]), however, which are beyond the scope of this thesis. The remainder of this chapter is organized as follows. The system model is presented in Section 6.2. HetNets with BF under the fully correlated fading condition are considered in Section 6.3 where the coverage probability is derived after the discussion of the impact of BF on the tier interference. Section 6.4 considers TD applied in HetNets under the fully correlated fading condition. Notice that the proposed analytical results are applicable for both types of HetNets, that is, the hybrid-modeled and Poisson point process (PPP)-modeled HetNets. Numerical results are presented in Section 6.5 with performance comparison of the two types of HetNets and the two type of multiple antenna techniques. Finally, Section 6.6 summarizes this chapter.

6.2 System Model

A downlink two-tier HetNet in the unlimited Euclidean plane where each BS is configured with multiple transmit antennas is considered in this chapter, characterized by the assumptions as stated in Section 3.2 of Chapter 3, with modifications of (A1) to (A4) included here to consider macro BSs with both types of deployments and the introduction of multiple antennas.

(A1) BS deployment: Both PPP-modeled HetNet and hybrid-modeled HetNet models are considered and compared in this chapter. In a PPP-modeled HetNet, the locations of BSs in each tier j are modeled by a PPP $\phi_j$ in the unlimited Euclidean plane, as stated in (A1) in Chapter 3. In a hybrid-modeled HetNet, small BSs are randomly dropped but macro BSs are hexagonal tessellated, as stated in (A1) in Chapter 4.
(A2) **Spectrum allocation:** For each tier $j$ with PPP modeled BS locations, the frequency groups $\rho_j$ are assumed to be randomly and independently allocated, since the regular pattern is not possible in a random BS deployment [14] [17]. However, for the hybrid model, macro BSs of the first tier are hexagonal tessellated and frequency are reused in the regular pattern [97], as shown in Fig. 4.2 where $\rho_1 = 3$ is assumed for demonstration.

(A3) **Power and transmission probability:** here is the same as stated in (A3) in Section 3.2, besides the further explanation that the transmit power $P_j$ at a BS in the tier $j$ is the total transmit power over multiple antennas at the BS.

(A4) **Propagation model and multiple antennas:** the transmitted signals in the tier $j$ experience log-distance path loss with the exponent $\alpha_j > 2$ [67] [97], and the received average power $\bar{h}_{j,i}$ from a BS $i$ in the tier $j$ at the distance $r_{j,i}$ is mathematically

$$\bar{h}_{j,i} = P_j r_{j,i}^{-\alpha_j} \quad (6.1)$$

There are $M_j$ omnidirectional antennas configured for each BS in the tier $j$, but all users are equipped with single omnidirectional antenna receiver, since the industry remains largely ambivalent about multiple antennas at user devices due to its strict complexity requirements [166] [168]. The signal from each antenna is Rayleigh faded implying the fading power gain is exponentially distributed. BF under the fully correlated fading case is considered in Section 6.3, where multiple antennas of each BS form a uniformly spaced linear phased array [97] [169] with a random direction and half a wavelength spacing among antennas. TD under the fully independent fading condition is studied in Section 6.4 where the fading is Rayleigh and independent among the $M_j$ antennas at the BS $i$. The
interference limited situation without intra-cell interference is considered, as stated in (A4) in Section 3.2.

6.3 Performance Analysis of HetNets with BF under fully correlated Fading Condition

All the analytical results involved with the same association policy are identical with those derived in Chapter 4, listed in Table 4.2 for the hybrid model and in Table 4.3 for the PPP model. Except these identical, the introduction of multiple antennas changes the statistics of the interference and the distribution of the desired signal, so does the coverage probability. Thus, the Laplace transform of the tier interference with BF is first derived in this section. After that, the coverage probability is formulated. The spectral efficiency can be achieved as that in Section 3.3.4 which is not repeated in this section.

6.3.1 Laplace transform of tier interference with BF under fully correlated fading condition

For Rayleigh fading fully correlated among multiple antennas, the total instantaneous received power by the considered user from all $M_j$ antennas at a BS $i$ in the tier $j$ is the multiplicative effects of the BF power spectrum $[97] [169]$ and Rayleigh fading power gain due to the independence between them. Mathematically, with consideration of BF applied with the uniformly spaced linear phased array, the total instantaneous received power by the considered user from all $M_j$ antennas at the BS $i$ in the tier $j$ becomes

$$h_{j,i} = \bar{h}_{j,i}X^\text{BF}$$

(6.2)

where $X^\text{BF}$ is the product of the fading power gain $X$, exponentially distributed with mean $E(X) = 1$ due to Rayleigh fading, and the BF power spectrum $BF_p(\psi)$ with $M_j$ antennas, which is presented in $[97] [169]$ and shown in (6.4). The wavenumber (the phase difference between transmit signals from neighboring antennas in terms of the direction to the considered user) $\psi = \pi(sin\theta_1 - sin\theta)$
where $\theta \in (-\pi, \pi]$ denotes the electronic steering direction to the serving user of the BS $i$, and $\theta_1 \in (-\pi, \pi]$ is the departure direction of the BS to the considered user.

$$X^{BF} = X \cdot BF_p(\psi)$$  \hspace{1cm} (6.3)

$$BF_p(\psi) = \frac{1}{M_j} \left( \frac{\sin \left( \frac{M_j \psi}{2} \right)}{\sin \left( \frac{\psi}{2} \right)} \right)^2 = \frac{1 - \cos(M_j \psi)}{M_j(1 - \cos \psi)}$$  \hspace{1cm} (6.4)

For an interfering BS not serving the considered user, $\theta_1$ and $\theta$ are independent of each other; while for the serving BS to the considered user, the steering direction $\theta_1$ is identical to the departure direction $\theta$ to the considered user, $\theta_1 = \theta$, thus $BF_p(\psi = 0) = M_j$ which represents the array gain.

**Lemma 6.1:** The locations of the interfering BSs each configured with $M_j$ antennas in tier $j$ are modeled by a PPP $\phi_j$ with intensity $\lambda_j$ in an unlimited plane excluding the area $b(0, d_j)$, the interference free disc of radius $d_j$ around the considered user. When BF is applied under Rayleigh fading fully correlated among multiple antennas, the Laplace transform of the aggregate interference $I_j$ received by the considered user is

$$L_{I_j}(s) = \exp \left\{-\pi \lambda_j \int_0^{2\pi} \int_0^{d_j} \left( 1 + \frac{a_j M_j (1 - \cos \psi)}{s P_j [1 - \cos(M_j \psi)]} \right)^{-1} f_\phi(\psi) \, d\psi \, dy \right\}$$  \hspace{1cm} (6.5)

where $\psi = \pi (\sin \theta_1 - \sin \theta)$, $\theta_1$ and $\theta$ are both uniformly distributed over the range $(-\pi, \pi]$ and independent with each other.

**Proof:** Provided in Appendix A3.1.

### 6.3.2 Coverage probability with BF under fully correlated fading condition

**Proposition 6.1:** For the 2-tier HetNet with fully correlated Rayleigh fading among multiple antennas and BF is applied, the coverage probability for a user at $r_1$ associating to the first tier is
\[ p_{c,l=1}(T|r_1) = e^{-\sum_{j=1}^{2} v_{lj} \left( \frac{Tr^{\alpha_i}}{M_P l} \right)} \]  
(6.6)

And the coverage probability for a user at \( r_1 \) associating to the second tier is

\[ p_{c,l=2}(T|r_1) = \int_0^{R_2} L_{I_{1,0}} \left( \frac{Tr^{\alpha_i}}{M_P l} \right) e^{-\sum_{j=1}^{2} v_{lj} \left( \frac{Tr^{\alpha_i}}{M_P l} \right)} f_{r_2}(r_2) dr_2 \]  
(6.7)

where in (6.6) and (6.7), \( f_{r_2}(r_2) \) is given in (4.10), and

\[ L_{I_{1,0}}(s) = 1 - \frac{\eta_1}{\rho_1} \int_{\psi} \left( 1 + \frac{r_1^{\alpha_j M_j (1-\cos \psi)}}{sP_1 [1-\cos(M_j \psi)]} \right)^{-1} f_\psi(\psi) d\psi \]  
(6.8)

\[ v_{lj}(s) = \pi \lambda_j \int_{d_{lj}}^{\infty} \int_{\psi} \left( 1 + \frac{y^{\alpha_j M_j (1-\cos \psi)}}{sP_j [1-\cos(M_j \psi)]} \right)^{-1} f_\psi(\psi) \, d\psi \, dy \]  
(6.9)

The subscript \( l \) denotes the associated tier index and \( j \) the tier index for the interference. \( d_{lj} \) is the radius of the interference-free disc in terms of the interference from tier \( j \) for users associating to tier \( l \), which is formulated in Table 4.2 for the hybrid model and Table 4.3 for the PPP model.

**Proof:** Provided in Appendix A3.2.

### 6.4 Performance Analysis of HetNets with TD under Independent Fading Condition

This section considers the independent fading condition among multiple antennas where TD is applied. The impact of TD on the tier interference Laplace transform is characterized first, then the coverage probability with TD is achieved. The received signal from each antenna at a BS \( i \) in the tier \( j \) to the considered user is independently Rayleigh faded, and then the received power \( X^{TD} \) at the considered user from all the \( M_j \) co-located antennas at the BS \( i \) is, as stated in [116], follows the Gamma distribution, \( X^{TD} \sim \gamma(M_j, \theta_\gamma) \), where the shape parameter is \( M_j \), and the scale parameter \( \theta_\gamma = 1/M_j \) due to the power normalization: \( E(X^{TD}) = M_j \times \theta_\gamma = 1 \). Notice that this conclusion
holds for both the desired signal and the interfering signals for the considered user. This is equivalent to the system with one single transmit antenna at each BS under Nakagami-\(m\) fading, since the power of a Nakagami-\(m\) faded signal follows the Gamma distribution with Nakagami parameter \(M_j\) [116].

### 6.4.1 Laplace transform of tier interference with TD under independent fading condition

**Lemma 6.2:** The locations of the interfering BSs each with \(M_j\) antennas in the tier \(j\) are modeled by a PPP \(\phi_j\) with intensity \(\lambda_j\) in an unlimited plane excluding the area \(b(0, d_j)\), the interference free disc of radius \(d_j\) around the considered user. When TD is applied with \(M_j\) antennas under independent Rayleigh fading, the Laplace transform of the aggregate interference \(I_j\) received by the considered user is

\[
\mathcal{L}_{I_j}(s) = \exp \left\{ -\pi \lambda_j s^{\frac{2}{\alpha_j}} \int_{\frac{2}{\alpha_j} d_j^2}^\infty \left[ 1 - \left( 1 + \frac{P_j y^{-\frac{2}{\alpha_j}}}{M_j} \right)^{-M_j} \right] dy \right\}
\]

(6.10)

**Proof:** Provided in Appendix A3.3.

When \(M = 1\), both (6.5) and (6.10) is simplified as follows, consistent with the previous result in [20] which is also shown in (3.16).

\[
\mathcal{L}_{I_j}(s) = e^{-2\pi \lambda_j d_j^2 \gamma s^{\frac{a_j}{\alpha_j}} \gamma^{-\frac{2}{\gamma}} \Gamma\left[1-\frac{2+a_j}{\alpha_j},-\frac{2}{\alpha_j} d_j^2 s^{\gamma}\right]}
\]

(6.11)

### 6.4.2 Coverage probability with TD under independent fading condition

**Proposition 6.2:** For the 2-tier HetNet with \(M_j\) antennas under independent Rayleigh fading, when TD is applied, the coverage probability for a user at \(r_1\) associating to the first tier is

\[
p_{c, l=1}(T|r_1) = \sum_{l=0}^{M_l-1} \frac{s^l}{l!} (-1)^l \frac{d^l}{ds^l} \left( e^{-\sum_{j=1}^{2} \psi_j(s)} \right)
\]

(6.12)
And the coverage probability for a user at \( r_1 \) associating to the second tier is

\[
p_{c, l=2}(T|r_1) = f_0^{R_2} \sum_{i=0}^{M_{l-1}} \frac{1}{i!} (-1)^i \frac{d^i}{ds^i} \left( L_{1,0}(s) e^{-\sum_{j=1}^{M_1} v_{ij}(s)} \right) f_{r_2}(r_2) dr_2
\]  

(6.13)

where in (6.12) and (6.13), \( f_{r_2}(r_2) \) is formulated in (4.10), and

\[
v_{ij}(s) = \pi \lambda_j \int_{d_{ij}^2}^{\infty} \left[ 1 - \left( 1 + \frac{s P_j y - \frac{a_j}{M_j}}{M_j} \right)^{-M_j} \right] dy
\]  

(6.14)

\[
L_{1,0}(s) = 1 - \eta_1 + \eta_1 \left( 1 + \frac{s P_1 r_1^{\alpha_1}}{M_j} \right)^{-M_j}
\]  

(6.15)

\[
s = \frac{M_1 r_1^{\alpha_1} T}{P_l}
\]  

(6.16)

**Proof:** Provided in Appendix A3.4.

Proposition 6.2 provides the exact analytical results, however, the high order derivatives are involved and very complicated to calculate. As presented in Proposition 3.1, the Gamma distribution function \( f_{\gamma}(x; M_l, 1/M_l) \triangleq \frac{x^{M_l - 1} e^{-x/M_l}}{\Gamma(M_l)} \) for the desired signal power can also be expressed by an exponential series as follows,

\[
f_{\gamma}(x; M_l, 1) \approx \sum_{n=1}^{N} w_n u_n e^{-u_n x}
\]  

(6.17)

Thus the Gamma cumulative distribution function \( F_{\gamma}(x; M_l, 1/M_l) \) is approximated by

\[
F_{\gamma}(x; M_l, 1) \approx 1 - \sum_{n=1}^{N} w_n e^{-u_n x}
\]  

(6.18)

whose coefficients \( w_n \) and \( u_n \) are determined as shown in Section 3.3.1.B.

Based on this exponential-series approximation, the coverage probability for the considered user at the distance \( r_l \) from the serving BS in the \( l^{th} \) tier is given as follows instead of (A3.4.1).

\[
p_{c, l}(T|r_l) = \sum_{n=1}^{N} w_n \left\{ \prod_{j=1}^{2} L_{lj}(u_n T r_l^{\alpha_l} P_l^{-1}) \right\}
\]  

(6.19)
Finally, by using (6.19) instead of (A3.4.1) and following the other steps of derivation steps from Proposition 6.2, Proposition 6.3 is achieved.

**Proposition 6.3:** For the 2-tier HetNet with independent Rayleigh fading among multiple antennas, when TD is applied, the coverage probability for a user at \( r_1 \) associating to the first tier is approximately

\[
p_{c,l=1}(T|r_1) = \sum_{n=1}^{N} w_n \left\{ \exp\left\{ - \sum_{j=1}^{2} v_{ij} \left( u_n T r_l^{a_l} p_l^{-1} \right) \right\} \right\}
\]  

(6.20)

And the coverage probability for a user at \( r_1 \) associating to the second tier is approximately

\[
p_{c,l=2}(T|r_1) = \int_{0}^{R_2} \sum_{n=1}^{N} w_n \left\{ L_{I_{1,0}} \left( u_n T r_l^{a_l} p_l^{-1} \right) \exp\left\{ - \sum_{j=1}^{2} v_{ij} \left( u_n T r_l^{a_l} p_l^{-1} \right) \right\} \right\} f_{r_2}(r_2) dr_2
\]  

(6.21)

where in (6.20) and (6.21), \( v_{ij}(s) \) is given in (6.14), \( L_{I_{1,0}}(s) \) in (6.15), and \( f_{r_2}(r_2) \) in (4.10).

As shown in Section 6.5, the proposed approximation with \( N \leq 8 \) provides a good accuracy. And the involved values of the coefficients \( w_n \) and \( u_n \) are listed in Table 6.2.

**6.5 Numerical Results and Discussion**

This section presents numerical results to demonstrate the performance benefits of multiple antennas exploited in 2-tier HetNets and to discuss the performance trend of HetNets with increasing number of antennas under different situations. Except noted, the assumed values of parameters used in the numerical evaluation are shown in Table 6.1, where the channel parameters are mainly borrowed from [21] and [67], and the subscripts “1” and “2” denote the macro BS tier and the small BS tier, respectively.
### Table 6.1 Assumed System Parameter Values [21] [67]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Assumed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_M$</td>
<td>Apothem of hexagon for the hybrid model; Equivalent apothem of hexagon for the PPP with intensity $\lambda_1$</td>
<td>1500m (leading to $\lambda_1 = 0.128/km^2$)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Intensity of macro BSs for the PPP model; Equivalent intensity of macro BSs for the hybrid model</td>
<td>$0.128/km^2$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Intensity of small BSs</td>
<td>$10 \times 0.128/km^2$</td>
</tr>
<tr>
<td>$P = [P_1, P_2]$</td>
<td>BS transmit power of tiers</td>
<td>[46, 26]dBm</td>
</tr>
<tr>
<td>$\alpha = [\alpha_1, \alpha_2]$</td>
<td>Path loss exponent of tiers</td>
<td>[3.76, 3.76]</td>
</tr>
<tr>
<td>$B = [B_1, B_2]$</td>
<td>Association bias of tiers</td>
<td>[1, 2]</td>
</tr>
<tr>
<td>$M = [M_1, M_2]$</td>
<td>Number of antennas in each tier</td>
<td>[4, 2]</td>
</tr>
<tr>
<td>$\rho = [\rho_1, \rho_2]$</td>
<td>Frequency reuse factor</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>$\eta = [\eta_1, \eta_2]$</td>
<td>Transmission probability</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>$G$</td>
<td>SINR gap</td>
<td>3dB</td>
</tr>
</tbody>
</table>

6.5.1 Validating proposed analysis through simulation

The analytical results are validated by Monte Carlo simulation in Matlab which is consistent with the system description in Section 6.2. The details of the simulation follow the depiction given in Section 4.4.1, except the steps (B1), (B2), and (B4) with some differences outlined as follows.

(B1) Both hybrid-modeled and PPP-modeled HetNet are considered as stated by (B1) in Section 5.4.1.

(B2) Location of the considered user is generated as stated by (B2) in Section 5.4.1.

(B3) The received power of the desired signal from multiple antennas at the associated BS is calculated which experiences path loss and Rayleigh fading, according to the situation for BF or TD.
Interfering signals from multiple antennas of each BS in the same frequency group and transmission slot are calculated based on the considered situation on path loss and Rayleigh fading for BF or TD.

The simulation results under different situations are represented by marks, compared with the analytical results embodied by curves in the following figures from Fig. 6.1 to Fig. 6.3, both are almost overlapped with each other, thus confirming the accuracy of the proposed analysis in this chapter. Notice that the analytical results for BSs each with single antenna in both tiers of the HetNet, derived in Chapter 4 and denoted as [1, 1] or SISO are also presented in Sections 6.5.3 and 6.5.4 for comparison.

### Table 6.2 Coefficients of exponential-series approximation for $\gamma(M, 1/M)$

<table>
<thead>
<tr>
<th>$n = 1$</th>
<th>$M = 2$</th>
<th>$M = 4$</th>
<th>$M = 8$</th>
<th>$M = 16$</th>
<th>$M = 64$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cdots, N$</td>
<td>$w_n$</td>
<td>$u_n$</td>
<td>$w_n$</td>
<td>$u_n$</td>
<td>$w_n$</td>
</tr>
<tr>
<td>1</td>
<td>84.505</td>
<td>2.0742</td>
<td>4.4854</td>
<td>136.50</td>
<td>9.4893</td>
</tr>
<tr>
<td>3</td>
<td>165.49</td>
<td>2.0035</td>
<td>-465.70</td>
<td>3.7618</td>
<td>72.593</td>
</tr>
<tr>
<td>4</td>
<td>-211.39</td>
<td>2.0251</td>
<td>-590.64</td>
<td>4.5997</td>
<td>-235.77</td>
</tr>
<tr>
<td>5</td>
<td>-234.65</td>
<td>5.1133</td>
<td>-241.17</td>
<td>12.075</td>
<td>-1409.9</td>
</tr>
<tr>
<td>6</td>
<td>-15.574</td>
<td>2.7599</td>
<td>277.47</td>
<td>7.5233</td>
<td>580.44</td>
</tr>
<tr>
<td>7</td>
<td>-48.471</td>
<td>2.9904</td>
<td>89.152</td>
<td>1.5828</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-304.96</td>
<td>25.7211</td>
<td>-370.77</td>
<td>7.3455</td>
<td></td>
</tr>
</tbody>
</table>

### 6.5.2 Performance comparison between hybrid model and PPP model

With BF applied under the fully correlated fading condition, the tier coverage probabilities of HetNets are compared between the hybrid model and the PPP model in Fig. 6.1 when the small BS to macro BS intensity ratio increases from $\lambda_2/\lambda_1 = 1$ to $\lambda_2/\lambda_1 = 100$. Both the hybrid model and
the PPP model have the same system parameters and the same antenna configuration with only one difference in macro BS deployment. When $\lambda_2/\lambda_1 = 1$, significant performance advantage of the hybrid model over the PPP model is demonstrated in Fig 6.1(a). The SIR threshold with 50% coverage probability in the hybrid model is 3.5dB higher than that in the PPP model (10.5dB vs 7.0dB) for the first tier, and 3.1dB higher (4.0dB vs 0.9dB) for the second tier. Correspondingly, the hybrid model with BF provides a 29% increase (3.47bits/s/Hz vs 2.69 bits/s/Hz) on the spectral efficiency of the first tier over the PPP model with BF, shown in Fig 6.1(c), and more than a 33% advantage (2.10bits/s/Hz vs 1.58bits/s/Hz) for the second tier, shown in Fig 6.1(d). Here is the reason: when macro BSs have the same intensity as small BSs, $\lambda_2/\lambda_1 = 1$, the dominant component of the interference comes from macro BSs with 20dB higher transmit power, thus the hexagonal tessellated macro BSs maximize the minimum distance between co-channel macro BSs and reduce the interference compared to randomly deployed macro BSs in the PPP model.

Now, the BSs in the second tier of the hybrid model and the PPP model are deployed in the same way, both are randomly deployed with the same antenna configuration. When small BS to macro BS intensity ratio increases to $\lambda_2/\lambda_1 = 10$ or even higher such as $\lambda_2/\lambda_1 = 100$, the interference component from randomly deployed small BSs accounts for the increasing proportion of the total interference and thus the difference between the interference of the hybrid model and the PPP model is falling, leading to the reduced performance advantage of the hybrid model.
(a) Tier coverage probability for $\lambda_2/\lambda_1 = 1$

(b) Tier coverage probability for $\lambda_2/\lambda_1 = 10$

(c) Spectral efficiency advantage of the hybrid model over the PPP model in 1st tier

(d) Spectral efficiency advantage of the hybrid model over the PPP model in 2nd tier

Fig. 6.1 Performance comparison between the hybrid model and the PPP model with increasing small BS to macro BS intensity ratio, $\lambda_2/\lambda_1$
As shown in Fig. 6.1(b), the difference of the SIR threshold with 50% coverage probability between
the hybrid model and the PPP model with BF is 0.8dB (9.1dB vs 8.3dB) for the first tier, and 1.0dB
for the second tier. Correspondingly the spectral efficiency benefits of the hybrid model provides
6.5% advantage (3.12bits/s/Hz vs 2.93 bits/s/Hz) on the spectral efficiency of the first tier over the
PPP model, shown in Fig 6.1(c), and about 10% advantage (1.93bits/s/Hz vs 1.75bits/s/Hz) for the
second tier, show in Fig 6.1(d).

For TD under the independent fading condition, the performance difference between the hybrid
model and the PPP model presents a similar trend as that for BF under the dependent fading
condition. Notice that the PDF of Gamma $\gamma(M_l, 1/M_l)$, where $M_l$ is the number of antennas at the
serving BS of the considered user, is approximated by the exponential series with the coefficients
$w_n$ and $u_n$, $n = 1, \cdots, N$, listed in Table 6.2 by solving the optimization problem as stated in Section
3.3.1.B. The number of terms of exponential series approximation is $N = 4$ for $M_l = 2$ and 4, $N =
6$ for $M_l = 8$, and $N = 8$ for $M_l = 16$ and 64. In short, as shown in Figs. 6.1(c) and 6.1(d), when
the intensity of the small BSs is comparable to or less than that of macro BSs, with multiple antennas
(including BF and TD) or single antenna at transmitter (SISO), the hybrid-modeled HetNet provides
much better performance for both tiers, compared to the PPP-modeled HetNets, while this
performance advantage decreases with the increasing intensity of small BSs and becomes marginal
when there are, on average, about 100 small BSs located in each hexagon. Since both types of
HetNets present similar performance trend and the difference between the two is not significant
when $\lambda_2/\lambda_1 = 10$ as considered in the following discussions, only the performance of BF in the
hybrid HetNet in Section 6.5.3 and TD in the PPP model in Section 6.5.4 are presented. Notice that
the performance of the second tier is lower than the first tier, mainly because the association bias
\[ B = [1,2], \] as listed in Table 6.1, is applied which pushes more users to associate to small BSs, though for these users the average received power from the closest macro BS is higher than that from its serving small BS.

**6.5.3 BF performance in hybrid-modeled HetNets**

By increasing the desired signal power at the considered user and reducing the interference to the other users, BF substantially boosts the coverage probability and spectral efficiency in the hybrid modeled HetNet under the fully correlated fading condition among multiple antennas. The median of the SIR statistics, which is equivalent with the SIR threshold with 50% coverage probability, is closely related to the average SIR and is used here to demonstrate the SIR improvement due to the array gain from BF.

As shown in Fig. 6.2(a) for the first tier, with single antenna at transmitter, the SIR statistic median is 3.0dB. When the number of antennas in both tiers increase to \( M = [8,8] \) or \( M = [64,64] \), the SIR statistic median increases to 12.8dB or 24.1dB, respectively. However, the coverage probability curves for different antenna configurations are almost parallel which means BF does not change the shape of SIR statistics. The enhancement of the location-dependent spectral efficiency with increasing number of antennas is shown in Fig. 6.2(b) for the first tier and Fig. 6.2(c) for the second tier. At the middle of the hexagon’s apothem, that is, \( r_1/R_M = 0.5 \), the spectral efficiency increases by 332% (from 1.79bits/s/Hz to 7.73bits/s/Hz) for the first tier, and 381% (from 1.71bits/s/Hz to 8.22bits/s/Hz) for the second tier, when the number of antennas for both tiers increases from \( M = [1,1] \) to \( M = [64,64] \). The tier spectral efficiency is presented in Fig. 6.2(d). The first tier provides the spectral efficiency of 1.91bits/s/Hz and the second tier 1.34bits/s/Hz for \( M = [1,1] \). When the number of antennas increases to \( M = [8,8] \) or \( M = [64,64] \), BF under the fully correlated fading
condition boost the spectral efficiency to 4.07bits/s/Hz or 7.27bits/s/Hz in the first tier, 3.43bits/s/Hz or 6.77bits/s/Hz in the second tier.

Fig. 6.2 Coverage probability and spectral efficiency in the hybrid modeled HetNet with beamforming, parameterized with increasing number of transmit antennas
6.5.4 Performance of TD and comparison with BF in PPP-modeled HetNets

TD delivers the full diversity gain to the PPP modeled HetNet under the independent fading condition among multiple antennas. As shown in Fig. 6.3(a), with the increasing number of antennas from [1,1] to [4,4], the curve of the coverage probability steepens and the coverage probability is considerably improved at the low SIR threshold $T \leq 0$dB, due to the diversity gain which lets the PDF of the SIR more concentrated around its average value. For the SIR threshold $T = -4$dB [28], the coverage probability increases from 77% to 89% when the number of antennas increases from [1,1] to [4,4]. But further increasing the number of antennas, such as [64,64] as shown in Fig. 6.3(a), does not improve the coverage probability considerably. Therefore, only 4 and less number of antennas configurations are considered in LTE for transmit diversity mode [167]. The coverage enhancement for the antenna configuration [8,8] is marginal compared to the antenna configuration [4,4] thus it is not shown in Fig. 6.3(a).

Though TD delivers the full diversity gain, TD does not considerably improve the average SIR, different from BF which provides the significant array gain. As shown in Fig. 6.3(a), when the number of antennas in both tiers increase from $M = [1,1]$ to $M = [4,4]$, the SIR statistic median increases only from 2.0dB to 2.8dB. If the number of antennas further increases to [64,64], the SIR statistic median marginally increases to 3.0dB. Without array gain achieved, TD delivers almost the same spectral efficiencies for different antenna configurations from [1,1] to [64,64] as shown in Fig. 6.3(b) for both tiers. The tier spectral efficiency is 1.67bits/s/Hz and 1.25bits/s/Hz for the first tier and the second tier at antenna configuration [1,1], which increases to 1.79bits/s/Hz and 1.32bits/s/Hz for both tiers at antenna configuration [4,4], and 1.81bits/s/Hz and 1.34bits/s/Hz at antenna configuration [64,64].
Fig. 6.3 Coverage probability and spectral efficiency of transmit diversity in the PPP modeled HetNet under the independent fading condition, compared to BF under the fully correlated fading condition.

The results for BF in the PPP modeled HetNet under the fully correlated fading condition are also presented in Figs. 6.3(a) and 6.3(b). As shown in Fig. 6.3(a), though BF does not deliver diversity gain as TD which offers higher coverage probability for the low SIR threshold (about $T < -5\text{dB}$), the achieved array gain from BF moves the coverage probability curve to the right on the whole which gives higher coverage probability than TD for the SIR threshold $T \geq -5\text{dB}$. At the same time, BF presents substantial advantage on spectral efficiency over TD with an increase of the antenna number. As shown in Fig. 6.3(b), at $M = [4,4]$, BF offers 66% more spectral efficiency (2.95bits/s/Hz vs 1.78bits/s/Hz) for the first tier, 78% more (2.33bits/s/Hz vs 1.31bits/s/Hz) for the second tier, compared to TD under the independent fading condition. The difference even increases...
further with the increase of antenna number. The spectral efficiency on the whole grows a little better than linearly with the antennas number in the logarithm scale with BF, while it is almost unchanged for TD.

6.6 Summary

An analytical approach is developed for performance analysis of 2-tier HetNets with BF under the fully correlated Rayleigh fading condition among multiple antennas and TD under the independent Rayleigh fading condition, successfully characterizes the impact of BF and TD on the interference and network performance, verified by simulation. Based on these analyses and numerical results, there are some key conclusions achieved. First, with multiple antennas, the hybrid-modeled HetNet provides much better coverage probability and spectral efficiency compared to the PPP-modeled HetNets, provided the intensity of small BSs is comparable to or less than that of macro BSs. When the intensity for both tiers is identical in a 2-tier HetNet with BF applied under the fully correlated Rayleigh fading condition, the hybrid model shows 29% improvement on spectral efficiency for the first tier and 33% improvement for the second tier compared to the PPP-modeled HetNet. This finding shows the importance of macro BS locations and the significance of network planning at the initial deployment phase of HetNets. The results also show that this performance improvement from the hybrid model decreases with the increasing intensity of small BSs and becomes marginal when there are, on average, about 100 small BSs located in each hexagon. Second, by concentrating the transmit power in the intended direction, BF indeed significantly boosts the coverage probability and spectral efficiency, and the spectral efficiency on the whole increases a little better than linearly with the number of antennas in the logarithm scale. Third, TD with 4 antennas delivers considerable improvement on the coverage probability, at the low SIR threshold, from 77% to 89%, at the SIR
threshold $T = -4 dB$, compared to the antenna configuration [1,1], however, the further increase of antenna number makes a marginal difference on the coverage probability. In terms of spectral efficiency, TD does not offer considerable improvement. Compared to TD under the independent fading condition delivering diversity gain to get the better coverage probability at the very low SIR threshold (about $T \leq -5 dB$), BF under the fully correlated fading condition provides impressive array gain which moves the coverage probability curve to right on the whole and provides the higher coverage probability than TD for the SIR threshold $T \geq -5 dB$. 
Chapter 7: Conclusions

In this thesis, the impact of four aspects in HetNets on interference statistics and the performance metrics is evaluated, and the analytical approaches are developed in the similar steps, forming an analytical framework to characterize the relationships of interference statistics and network performance with some key system design parameters. This chapter summarizes the major research findings achieved by this analytical framework, followed by the thesis conclusions, the engineering significance of findings and conclusions, and suggestions on future work.

7.1 Major Research Findings

According to the four aspects of HetNets considered in this thesis, the major findings in this research are summarized as follows.

7.1.1 Performance analysis of PPP-modeled HetNets with presence of LOS and Shadowing

The impact of the presence of LOS and shadowing on interference statistics of Poisson point process (PPP)-modeled HetNets is captured by its Laplace transform and the performance influence is derived in mathematical expressions in Chapter 3. Briefly speaking, when the inter-tier interference from macro BSs is Rayleigh faded and the small BS to macro BS intensity ratio \( \lambda_2/\lambda_1 \leq 10 \), the presence of LOS in the desired signal from small BSs has a significant enhancement in coverage probability when SIR threshold is less than 5dB, though the resulting improvement in spectral efficiency is marginal. For the interfering signals from small BSs, however, the impact of LOS existence is very marginal on both the coverage probability and spectral efficiency, under the assumption that the interfering signals from macro BS are Rayleigh faded. Contrary to the performance impact of the presence of LOS, shadowing deteriorates both the coverage probability...
and spectral efficiency significantly in HetNets with the Rayleigh fading and the biased association policy, especially in the range of the shadowing standard deviation $\sigma_{dB} > 4dB$.

In the analysis, the exponential-series approximation with only 4 to 8 terms gives acceptable accuracy on performance analysis for fitting the PDF of the signal power under Rician fading or lognormal shadowed Rayleigh fading.

### 7.1.2 Performance analysis of hybrid-modeled HetNets with hexagonal tessellated macro BSs

The interference from hexagonal tessellated macro BSs is verified with acceptable accuracy to an approximation generated by the shot noise interference model with an interference-free disc around the considered user under the lognormal shadowed Rayleigh fading. Based on this approximation, interference statistics and the performance metrics are derived for HetNets with hexagonal tessellated macro BSs and the significance of network planning for macro BS locations and frequency reuse is quantitatively evaluated under different situations. Looking into more details on location-dependent performance metrics, it is found that the spectral efficiency loss due to high frequency reuse factor is marginal or even better for a user far away from its serving BS in HetNets under Rayleigh fading, which provides the reasoning for location-dependent frequency reuse interference management strategies, such as FFR investigated in this thesis, to satisfy the stringent requirements on coverage probability and spectral efficiency. In HetNets, the tier of macro BSs is assumed for coverage and the tier of small BSs for capacity from the perspective of future radio access networks [1] [133]. If so, with increasingly dense small BSs, HetNets need to reserve dedicated resource to macro BSs in order to ensure that the whole service area is covered by macro BSs.
7.1.3 Performance analysis and partitioning optimization for FFR in HetNets

The developed analyses of location-dependent coverage probability and spectral efficiency presents more details for interference management in HetNets, and based on these, FFR in two-tier HetNets is evaluated which reveals the significant enhancement on the coverage probability of macro BSs in the edge region, in addition to macro BS interference reduction and small BS interference reduction for users in the second tier. Numerical results demonstrate that, in PPP-modeled HetNets with FFR utilized macro BSs, the coverage enhancement comes with the cost of lower system spectral efficiency, however, hybrid-modeled HetNets have potential to provide the improvement of system spectral efficiency in addition to the coverage enhancement with FFR. This different conclusion again highlights the difference between hybrid-modeled and PPP-modeled HetNets. FFR benefits in terms of system spectral efficiency in hybrid-modeled HetNets are identified and the dependence of the optimal partitioning of FFR regions on transmission probability, shadowing standard deviation, and the intensity of small BSs are investigated in detail in the thesis.

7.1.4 Performance analysis of HetNets with multiple antennas

Interference statistics are formulated by its Laplace transform and the performance is derived for 2-tier HetNets with BF under the fully correlated Rayleigh fading among multiple antennas and TD under the independent Rayleigh fading. The developed analyses reveal that, with multiple antennas, the hybrid-modeled HetNet provides much better coverage probability and spectral efficiency compared to the PPP-modeled HetNets, provided the intensity of small BSs is comparable to or less than that of macro BSs. The analytical results also show that, this performance improvement from the hybrid model decreases with the increasing intensity of small BSs and becomes marginal when there are, on average, about 100 small BSs located in each hexagon. Second, with consideration of
intra-tier and inter-tier interference, packing more antennas to BSs with BF indeed significantly boosts the coverage probability and spectral efficiency of HetNets. BF raises the spectral efficiency on the whole a little better than linearly with the number of antennas in the logarithm scale. Third, TD with 4 antennas delivers considerable improvement on the coverage probability at the low SIR threshold compared to the antenna configuration [1,1], however, the further increase of antenna number makes a marginal difference on the coverage probability. Moreover, TD does not offer considerable improvement in terms of spectral efficiency. Compared to TD under the independent fading condition delivering diversity gain to get the better coverage probability at the very low SIR threshold (about $T \leq -5dB$), BF under the fully correlated fading condition provides impressive array gain which moves the coverage probability curve to the right on the whole and provides the higher coverage probability than TD for the SIR threshold $T \geq -5$ dB. In the analysis, the exponential-series approximation with only 4 to 8 terms gives acceptable accuracy on performance analysis for fitting the PDF of the signal power with Gamma distribution and the shape parameter $M \leq 64$.

7.2 Thesis Conclusions

The interference in HetNets becomes one of the major obstacles to further capacity improvement and coverage enhancement, thus calls for effective interference management solutions to achieve the promising performance gain by deploying more small BSs. As shown in the developed analytical results in this thesis, the interference statistics and network performance depend on so many system parameters, such as propagation parameters, frequency reuse and power allocation, small BS intensity and macro BS intensity, transmission probability and SIR gap from Shannon capacity, and so on. Thus, one conclusion of this thesis is that HetNet designers and architects must watch out
whether or not a setup of system parameters is proper to a selected interference management strategy. As revealed in Chapters 4 and 6, when small BSs are not densely deployed, tessellated macro BSs significantly reduce the impact of interference thus considerably improve the performance for all users, including users associating to macro BSs and users associating to small BS. However, it is also shown that, the performance advantage of tessellated macro BSs is not significant any more in HetNets with dense small BSs, i.e., about 100 small BSs overlaid on each hexagon on average. As demonstrated in Chapter 5, the comprehensive investigation reveals that, when the interference is dominated by the interfering power from macro BSs, FFR is more suitable in terms of spectral efficiency for situations where the load level is relatively high and the standard deviation of shadowing is low. However, when the transmission probability is low, or the HetNet has densely deployed small BSs, or there exists severe shadowing, the advantage of FFR in terms of spectral efficiency becomes marginal or even disappears.

In this theses, the performance impact of the four aspects of HetNets is analyzed which shows several insights on interference management: (1) With the increasingly dense small BSs and careful BS deployment planning, the user transmitted signal experiences line of sight propagation condition more frequently with more nearby BSs, leading to reduced fading and shadowing variation and then improved performance on both coverage probability and spectral efficiency. (2) Macro BS deployment planning in practice indeed has its significance to mitigate the impact of the interference thus resulting in considerable performance benefit for users associating to both tiers in HetNets with lightly deployed small BSs. In contrast, with densely deployed small BSs, since dedicated resources are needed for macro BSs in order for them to provide the fully coverage over the service area of a HetNet, macro BS deployment planning thus still provides a significant performance enhancement
under this situation. (3) The developed location-dependent performance analyses reveal that, for a user close to its serving macro BS, the increase of the reuse factor improves the coverage probability, however, with the cost of a significantly reduced spectral efficiency at the system level. At the same time, for a user far away from its serving macro BS, the increase of reuse factor improves the coverage probability significantly, but with a cost of high reduction in the system spectral efficiency. This observation provides the rationale for location-dependent interference management strategies such as the FFR. (4) Transmit diversity with 4 or less antennas considerably reduces the received power variation due to independent Rayleigh fading and provides coverage enhancement at the low SIR threshold, while there is no advantage for further increase in the number of antennas. Different from transmit diversity, beamforming with increasing number of antennas continues to provide impressive performance boost on both coverage probability and spectral efficiency, by concentrating the energy in the intended direction and significantly mitigating the interference impact to other users.

In short, the interference statistics and network performance depend on these different factors of HetNets, and the impact of interference can be avoided or mitigated when each of these different aspects is considered. Overall, in practice, a holistic interference management scheme can be formed with a consideration of these various factors together for a HetNet during its evolution process in order to achieve the promising performance gain by deploying more small BSs.

7.3 Engineering Significance of Thesis Findings and Conclusions

Interference management is widely recognized as a major impairment limiting system performance in HetNets and requires efficient solutions for future radio access networks. In this context, the developed analytical framework in this thesis potentially paves a way to evaluate emerging
interference management strategies under different situations, and the characterized relationships of interference statistics and network performance with key network design parameters in mathematical expressions help inspire an in-depth understanding in interference management for HetNets. Through the research findings summarized in Section 7.1, several concerns related to the design of HetNets are addressed and some insights on interference management are reported, contributing to the existing body of knowledge. It is also expected that the analyses and the discussion in this thesis are able to highlight the necessity of alternative and advanced approaches to mitigate the interference impact and enhance the network performance in a more comprehensive manner, such as advanced signal processing techniques (that is, advanced spatial multiplexing techniques, advanced receivers with interference cancellation ability), and channel-aware scheduling techniques, discussed in detail in Section 7.4.

7.4 Thesis Limitations and Suggestions for Future Work

7.4.1 Thesis limitations

As stated in [14], interference statistics and network performance analysis have always been a challenging problem even in simple traditional single-tier cellular networks. For the research in this thesis, there exist two key limitations.

1. The analytical framework developed in the thesis is limited to the cases with the independence of interfering BSs. When some kinds of correlation among these uncertainties are introduced, the analysis becomes much harder with the loss of additional level of analytical tractability, as shown in [170].

2. The analytical results in the thesis are achieved in a statistical way, especially for PPP-modeled HetNets. The overall network performance may not necessarily be the scaled version of the per-
cell performance that varies from BS to BS depending on their traffic load and neighboring BS deployment. Thus the worst cases in HetNets are also expected to be investigated and solved in order to provide the mobile data services to users ubiquitously and transparently [3].

7.4.2 Suggestions for future work

As a logical extension of the work presented in this thesis, the future study items are recommended as follows.

A. *Interference management and performance analysis of HetNets with advanced signal processing techniques*

Given their increasing importance to enhance the network capacity and mitigate interference in future radio access networks, multiple antenna techniques are expected to be more extensively utilized [6] [123] and the performance impact of various advanced multiple antenna techniques is vitally expected to be analyzed on interference statistics and the performance in HetNets, such as multi-user multiple-input and multiple-output (MU-MIMO) [30], Coordinated Multi-Point (CoMP) [171], and Massive MIMO [6]. MU-MIMO and CoMP have the potential to provide spatial multiplexing gains in interference-limited cellular systems, and they are already included in the LTE-A standards. Massive MIMO, one of promising enablers for the next generation of cellular networks [3] [6], is a further evolution of multiple antenna techniques when many more antennas are deployed at BSs than users served by them. In addition to spatial multiplexing gains intensively investigated in the previous studies, these multiple antenna techniques utilized at the BS end actively avoid the interference to be generated, significantly reshape the interference statistics in HetNets, which is expected to be analyzed. Instead of these interference avoidance techniques at the BS end, passively interference cancellation and suppression techniques used at the advanced receiver end
[158] [172] [173], such as aforementioned interference rejection combining (IRC) [37] and successive interference cancellation (SIC) [38], to provide additional performance gain at the other end of signal transmission by allowing an existing receiver to operate with higher levels of co-channel interference, which is also expected to be analyzed. The results developed should be feasible to provide an accurate and elegant evaluation measure for the achievable performance gain, and how to balance the performance improvements and the implementation complexity as well as the increased signaling overhead in the air-interface. Based on the expected mathematical expressions to characterize the relationships of interference statistics and network performance with key network design parameters, it is also expected to help inspire more in-depth understanding on interference management in HetNets with consideration of these advanced signal processing techniques.

B. Interference management and performance analyses of HetNets using channel and queue state information

In the analyses presented in this thesis, the users are assumed to be uniformly distributed over the service area and thus the probability distribution function (PDF) of the distance to each associated BS is derived with consideration of the biased association policy. The developed analyses still hold if round robin scheduling is applied, since round robin scheduling does not change the PDF of the distance and the power statistics of the desired signal. Compared to round robin scheduling, variations in channel and queue state information among users in a BS can be exploited [174] [175] which provide multi-user diversity gain, such as proportional fair (PF) scheduling [176] or more generalized utility maximum algorithms [177]. Given the use of channel and queue state information in practical cellular networks to enhance the network capacity and to flexibly trade-off between spectral efficiency and fairness, these channel/queue-aware scheduling techniques with system level
coordination in HetNets are expected to be investigated and analyzed, which are challenging due to the complicated power distribution of the desired signal and the changed distance distribution of scheduled user locations. Furthermore, in addition to data awareness, service awareness, environmental awareness, and social and economic awareness are also expected to be considered in HetNets regarding the research and development of future network technologies [178].

C. Mobility management in HetNets

Mobility management, which comes as another technical challenge with HetNets, calls for new approaches [1] [2] [3]. In HetNets, the more small BSs are packed into a given area, the higher the interference which needs to be mitigated. At the same time, users move across cell-edges more frequently. For example, the user needs to move from one cell to the other in every few seconds with a coverage area of diameter under 100m and at the velocity of 30km/h [2]. Studies in long term evolution (LTE) show that there are even more frequency handover events with nearby small BSs in the shared spectrum [179]. Under these situations, a handover mechanism based only on the received signal strength is not efficient and robust, and the analysis indicates that handover performance in HetNet deployments is not as good as in pure macro [2]. Thus some advanced mobility management schemes are proposed in the literature. One simple scheme is user grouping based on their velocities, where stationary users are served by small BSs while mobile users are allocated to the macro BSs [180] [181]. Another mobility management scheme is clustering nearby small BSs as a single virtual BS and handovers are triggered only at cluster boundaries [2]. Recently, some analytical approaches for mobility management are also developed. For example, the impact of mobility on handover rate and sojourn time is analyzed in a homogenous network based on a proposed random waypoint mobility model [182], and a stochastic geometric analysis framework
[183] is developed to evaluate rates of different types of handover in PPP-modeled HetNets, which provides guidelines for optimal tier selection with consideration of both the handoff rates and the data rate. Though these studies provide some insights and guidelines for mobility management in HetNets, however, they are based on oversimplified system models. More diverse characteristics of HetNets, such as access policy, bandwidth, latency, cost, coverage, and quality of service etc., lead to complicated situations which call for new effective solutions for robust and reliable mobility management at low cost [184], in order to provide features such as “best experience follows you” [3].
References


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A1 Appendices to Chapter 3

A1.1 Proof of Proposition 3.1

The main idea of the proof is to show that \( g(x) \) is equivalent to a generalized polynomial, thus \( g(x) \) with an increasing number of terms has the potential to converge to \( f_x(x) \). Denoting \( z = e^{-x} \) and \( w'_n = w_n u_n \), then the composite function \( g(-\ln z) = \sum_{n=1}^{N} w'_n z^{u_n} \). Now a special case is constructed where \( u_n = n \). For this special case, \( g(-\ln z) \) becomes a polynomial of degree \( N \), and it is trivial to prove \( f_x(-\ln z) \) is a continuous real-valued function on the real interval \( z \in [e^{-V}, 1] \). Then by the Stone–Weierstrass theorem [185], for any small value \( \varepsilon > 0 \), there exists a \( w'_n \) such that for all \( z \in [e^{-V}, 1] \), \( |f_x(-\ln z) - g(-\ln z)| < \varepsilon \). Equivalently, there exists the coefficients \( u_n \geq 0 \) and \( w_n = w'_n / u_n \) making \( |f_x(x) - g(x)| < \varepsilon \) on the real interval \( x = -\ln(z) \in [0, V] \). □

A1.2 Proof of Lemma 3.1

A serving area \( A_{j,k} \) is defined as an annular area around the considered user with the inner radius of \( d_j \) and the outer radius \( R_k > d_j, k = 1, 2, \ldots, \forall k, R_k < R_{k+1} \), and \( \lim_{k \to \infty} R_k = \infty \) which means the annular area tends to an unlimited plane. For PPP \( \phi_j \backslash b(0, d_j) \) in the serving area \( A_{j,k} \), the number of points \( n \) is Poisson distributed with the intensity of \( \lambda_j A_{j,k} \). Thus, the Laplace transform of \( I_j \) is

\[
L_i(s) = E_i(e^{-sl}) = E_i(\phi_j \backslash b(0, d_j)) \left\{ \exp \left[ -s \sum_{i \in \phi_j \backslash b(0, d_j)} (P_j X_{ji} r_{ji}^{-\alpha_j}) \right] \right\}
\]

\[
\overset{(a)}{=} \lim_{k \to \infty} E_{i \in \phi_j \in A_k \backslash b(0, d_j)} \left\{ \exp \left[ -s \sum_{i \in \phi_j \in A_k \backslash b(0, d_j)} (P_j X_{ji} r_{ji}^{-\alpha_j}) \right] \right\}
\]

\[
\overset{(b)}{=} \lim_{k \to \infty} E_{i \in \phi_j \in A_k \backslash b(0, d_j)} \sum_{n=0}^{\infty} P[n] \left( E_{X,r} \left[ \exp \left( -s P_t X r^{-\alpha_j} \right) \right] \right)^n
\]  

(A1.2.1)
where the series in (a) converges to $L_I(s)$ since the series is monotonic and bounded, $P[k]$ in (b) is the probability mass function of Poisson-distributed random variable $n$ with the parameter $\lambda_j A_{j,k}$.

Due to the independence of points in PPP $\phi_j$ and the independent, identically distributed fading among the interfering signals, all the subscripts of interfering BSs in (A1.2.1) are dropped out. By substituting the probability mass function of Poisson distribution $P[n]$ into (A1.2.1),

$$L_I(s) = \lim_{k \to \infty} E_i \in \phi_j \in A_k \setminus b(0,d_j) \sum_{n=0}^{\infty} \left( \frac{(\lambda_j A_{j,k})^n}{n!} \left( E_{X,r}[\exp(-s P_i X r^{-\alpha_j})] \right) \right)^n$$

$$= \lim_{k \to \infty} E_i \in \phi_j \in A_k \setminus b(0,d_j) \exp \left( \lambda_j A_{j,k} \left( E_{X,r}[\exp(-s P_j X r^{-\alpha_j})] - 1 \right) \right)$$

$$(c) \quad \exp \left\{ - \lim_{k \to \infty} \int_{A_{j,k}} \left[ 1 - E_{X}[\exp(-s P_j X r^{-\alpha_j})] \right] \lambda_j dA \right\}$$

$$(d) \quad \exp \left\{ - 2\pi \lambda_j \int_{d_j}^{R_k} \left[ 1 - L_X(s P_j r^{-\alpha_j}) \right] 2\pi \lambda_j r dr \right\}$$

$$= \exp \left\{ - 2\pi \lambda_j \int_{d_j}^{\infty} \left[ 1 - L_X(s P_j r^{-\alpha_j}) \right] r dr \right\} \quad (A1.2.2)$$

where using the integration over the serving area and the continuity of exponential function give (c) and integration in the polar coordinate gives (d). Note that $r$ is the distance of element area $dA$ from the considered user in (c), $L_X(\cdot)$ is the Laplace transform of the random variable $X$, the power of a interfering signal under Rician fading, which follows the scaled non-central chi-squared distribution as $X' = (2 + 2K_I)X \sim \chi^2_{k=2}(\lambda = 2K_I)$, whose Laplace transform is

$$E_{X'}[\exp(-sX')] = \frac{\exp\left(-\frac{\lambda_{2K_I}}{1+2s}\right)}{(1+2s)\lambda_{2K_I}} = \frac{\exp\left(-\frac{2K_I s}{1+2s}\right)}{1+2s} \quad \text{for } s > -\frac{1}{2} \quad (A1.2.3)$$

Since $L_X((2 + 2K_I)s) = E_X[\exp(-(2 + 2K_I)sX)] = E_{X'}[\exp(-sX')]$,

$$L_X(s) = \frac{(1+K_I)\exp\left(-\frac{K_I s}{1+K_I+s}\right)}{1+K_I+s} \quad (A1.2.4)$$
Then substituting (A1.2.4) into (A1.2.2) gives

\[ \mathcal{L}_{ij}(s) = \exp \left\{ -2\pi \lambda_j \int_{d_j}^{\infty} \left[ 1 - \mathcal{L}_X \left( sP_j r^{-\alpha_j} \right) \right] r \, dr \right\} \]

\[ = \exp \left\{ -2\pi \lambda_j \int_{d_j}^{\infty} \left[ 1 - \frac{(1+K_I) \exp \left( \frac{-K_I sP_j r^{-\alpha_j}}{1+K_I sP_j r^{-\alpha_j}} \right)}{1+K_I sP_j r^{-\alpha_j}} \right] r \, dr \right\} \]

\[ (e) \quad = \exp \left\{ -2\pi \lambda_j \int_{d_j}^{\infty} \left[ 1 - \frac{r_{ij}^{\alpha_j} \exp \left( \frac{-vK_I}{v+r_{ij}^{\alpha_j}} \right)}{v+r_{ij}^{\alpha_j}} \right] r \, dr \right\} \]

\[ (f) \quad = \exp \left\{ -2\pi \lambda_j \int_{d_j+1}^{\infty} \left[ 1 - \frac{(y-1) \exp \left( \frac{-K_I}{y} \right)}{y} \right] \left[ v(y-1) \frac{1}{\alpha_j[y(y-1)]^{-\alpha_j}} \right] \right\} \]

\[ (g) \quad = \exp \left\{ -2\pi \lambda_j \frac{v^{\alpha_j}}{\alpha_j} \int_{d_j+v}^{\infty} \left[ 1 - (1-z) \exp(-K_I z) \right] z^{-\alpha_j} \left( 1 - z \right)^{2\alpha_j-2} \, dz \right\} \]

\[ = \exp \left\{ -2\pi \lambda_j \frac{v^{\alpha_j}}{\alpha_j} \int_{d_j+v}^{\infty} \left[ 1 \right] z^{-\alpha_j} \left( 1 - z \right)^{2\alpha_j-2} \, dz \right\} \quad \text{(A1.2.5)} \]

where the variable substitution \( v = \frac{sP_j}{1+K_I} \) is used in (e), \( y = \frac{r_{ij}^{\alpha_j}}{v} + 1 \) in (f), and \( z = \frac{1}{y} \) in (g).

Further using the Taylor series expansion for the exponential function in (A1.2.5) gives the proof as follows

\[ \mathcal{L}_{ij}(s) = \exp \left\{ -2\pi \lambda_j \frac{v^{\alpha_j}}{\alpha_j} \left[ \int_{d_j+v}^{\infty} z^{-\alpha_j} \left( 1 - z \right)^{2\alpha_j-2} \, dz - \sum_{n=1}^{\infty} \frac{(-K_I)^n}{n!} \int_{d_j+v}^{\infty} z^{-\alpha_j} \left( 1 - z \right)^{2\alpha_j-2} \, dz \right] \right\} \]

\[ = \exp \left\{ -2\pi \lambda_j \frac{v^{\alpha_j}}{\alpha_j} \left[ \sum_{n=0}^{\infty} (-1)^{n+l_{n \neq 0}} \frac{K_I^n}{n!} \frac{B}{\alpha_j^v} \left( n + l_{n=0} - \frac{2}{\alpha_j^v} + l_{n \neq 0} \right) \right] \right\} \quad \square \]
A1.3 Proof of Proposition 3.2

By definition, the coverage probability for the considered user at the distance $r_l$ from the serving BS in the $l^{th}$ tier is given by:

$$p_{c,l}(T|r_l) = P_r\left[\frac{h}{\sum_{j=1}^{l} I_j} > T|r_l\right] = P_r\left[P_l r_l^{-\alpha_l} X > \sum_{j=1}^{l} I_j T|r_l\right]$$

equivalent to

$$= E_{l_1,l_2}\left\{P_r \left[X > \frac{\alpha_l}{P_l} \left(\sum_{j=1}^{l} I_j\right) T|r_l, I_1, I_2\right]\right\}$$

$$= \sum_{n=1}^{N} w_n \left\{\prod_{j=1}^{2} L_{I_j}\left(u_n Tr_l^{-\alpha_l} P_l^{-1}\right)\right\}$$

where in (h), $h$ is the power gain of the desired signal fading in the $l^{th}$ tier. Note that the desired signal is Rician faded so that the power gain follows the non-central chi-squared distribution.

$$P_r[X > x] = \tilde{G}(x),$$

invoking the exponential series approximation (3.9) gives (i). In (j) $L_{I_j}(s) = E_{I_j}\{e^{-s I_j}\}$, the Laplace transform of the aggregate interference power $I_j$ from the $j^{th}$ tier.

As stated in the biased association policy (3.3), the considered user associates to the tier $l$ since

$$P_l r_l^{-\alpha_l} B_l \geq P_j r_j^{-\alpha_j} B_j, \forall j \in [1,2]$$

This implies that the distance from the considered user to anyone of the interfering BSs $i$ in the tier $j$ must satisfy

$$r_{j,i} \geq \tilde{r}_j \geq \left(\frac{P_j B_j}{P_l B_l}\right)^{\frac{1}{\alpha_j}}\frac{\alpha_j}{\alpha_l} r_l^{-\alpha_j} = d_j$$

where $d_j$ denotes the radius of the interference disc in the tier $j$ around the considered user.

The Laplace transform of $I_j$ is given by substituting (3.23) into (3.16),
\[ L_j(s) = e^{-2\pi \lambda_j \frac{\frac{2}{a_j} I_j^2}{a_j}} \sum_{n=0}^{\infty} (-1)^{n+I_n\neq 0} \frac{K_{1/2}}{n!} B(s) \left( n+I_n=0 - \frac{2}{a_j} + I_n=0 \right) \]  
(A1.3.4)

where \( x_j(s) = \frac{sB_i P_i}{(1+K_{ij}) B_j r_i + sB_i P_i} \).

Substituting (A1.3.4) into (A1.3.1) concludes the proof. \( \square \)

### A1.4 Proof of Proposition 3.3

The location-dependent spectral efficiency of a considered user at the distance \( r \) is \( \tau(r) \triangleq E[ln(1 + SINR/G) | r] \) where the average is taken over both the interfering PPP and the joint distribution of fading and shadowing. Since the power is positive so does \( \log_e (1 + SINR/G) \), thus for a positive random variable \( X, E[X] = \int_{z>0} P_r(X > z)dz \), so that

\[
\tau_l(r) = \int_{z>0} P_r \left[ \ln \left( 1 + \frac{SIR}{G} \right) > z | r \right] dz
\]

\[
= \int_{z>0} P_r[SIR > G(e^z - 1)|r]dz
\]

(A1.4.1)

Substituting \( T = G(e^z - 1) \) into (A1.4.1) gives

\[
\tau_l(r) = \int_{z>0} \frac{1}{T+G} P_r[SIR > T|r]dT
\]

\[
= \int_{z>0} \frac{1}{T+G} p_{c,l}(T|r)dT
\]

(A1.4.2)

In the same way, the tier spectral efficiency is

\[
\tau_l = \int_{z>0} P_r \left( \ln \left( 1 + \frac{SIR}{G} \right) > z \right) dz
\]

\[
= \int_{z>0} \frac{1}{T+G} P_r(SIR > T)dT
\]

\[
= \int_{z>0} \frac{1}{T+G} p_{c,l}(T)dT
\]

(A1.4.3)

This concludes the proof. \( \square \)
A1.5 Proof of Lemma 3.2

Similar to the proof of Lemma 3.1, the Laplace transform of \( I_j \) in (A1.2.2) holds for the interference with any distribution, including lognormal shadowed Rayleigh fading. Instead of (A1.2.3) where the derivation starts assuming Rician fading, the joint distribution of lognormal shadowed Rayleigh fading is seen as the distribution of Rayleigh fading superimposed with a known lognormal shadowing value \( \mu_I \), then its Laplace transform is

\[
L_g(s) = E_{\mu_I} \left\{ \frac{1}{1+\mu_I s} \right\} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{1}{1+sexp(\sqrt{2}a_Iy)} exp(-y^2) \, dy \tag{A1.5.1}
\]

The Gauss-Hermite quadrature in numerical analysis [66] is exploited in order to circumvent this layer of integration in (A1.5.1) and \( L_g(s) \) can be written as

\[
L_g(s) = \sum_{n=1}^{N} \omega_n \frac{1}{\sqrt{\pi}} \frac{1}{1+sexp(\sqrt{2}a_n)} + O_N \tag{A1.5.2}
\]

where \( O_N \) is a remainder term that decreases to zero as \( N \) increases to infinity. The weights \( \omega_n \) and abscissas \( a_n \) are determined by Hermite polynomial after \( N \) is determined. A small value of \( N \) is generally enough for approximation [66] and numerical results point out that \( N = 8 \) is sufficient for the present approach. In other words, when \( N \geq 8 \), \( O_N \approx 0 \).

Substituting (A1.5.2) into (A1.2.2) and combining with the identity \( \sum_{n=1}^{N} \omega_n = \sqrt{\pi} \) [66] gives

\[
L_1(s) = exp \left\{ -\pi \lambda_I \int_{d_j}^{\infty} \left[ \frac{\sum_{n=1}^{N} \omega_n}{\sqrt{\pi}} - \frac{\sum_{n=1}^{N} \omega_n}{\sqrt{\pi}} \frac{1}{1+sp_jy} \right] \exp(-\sqrt{2}a_n) \, dy \right\}
\]

\[
= exp \left\{ -\sqrt{\pi} \lambda_I \int_{d_j}^{\infty} \left[ \sum_{n=1}^{N} \frac{\omega_n a_n}{\sqrt{\pi}} \right] \exp(-\sqrt{2}a_n) \, dy \right\}
\]

\[
= exp \left\{ -\sqrt{\pi} \lambda_I \int_{d_j}^{\infty} \left[ \sum_{n=1}^{N} \frac{\omega_n a_n}{\sqrt{\pi}} \right] \exp(-\sqrt{2}a_n) \, dy \right\}
\]
\[
\begin{aligned}
&= \exp \left\{ -\sqrt{\pi} \lambda \left( sP_j \right)^{\frac{2}{\alpha_j}} \int_{-\infty}^{\infty} \left[ \sum_{n=1}^{N} \frac{\omega_n}{\alpha_j} \right] \frac{dy}{\left( y(sP_j)^{\frac{2}{\alpha_j}} \right) \exp(-\sqrt{2}\sigma_j a_n)} \right\} \\
&= \exp \left\{ -\sqrt{\pi} \lambda \left( sP_j \right)^{\frac{2}{\alpha_j}} \int_{-\infty}^{\infty} \left[ \sum_{n=1}^{N} \frac{\omega_n}{1+\nu^2} \exp(-\sqrt{2}\sigma_j a_n) \right] dv \right\} \\
&= \exp \left\{ -\sqrt{\pi} \lambda \left( sP_j \right)^{\frac{2}{\alpha_j}} \sum_{n=1}^{N} \omega_n \exp(2\sqrt{2}\sigma_j a_n/\alpha_j) \int_{-\infty}^{\infty} \frac{1}{\alpha_j} \left[ \frac{dv}{1+\nu^2} \right] \right\} (A1.5.3)
\end{aligned}
\]

where \( v = sP_j y \) in (k). Finally applying the integral identity (A1.5.4) [152] into (A1.5.3) gives (3.30).

\[
\int_{d}^{\infty} \left( \frac{1}{c} \right)^{d-\alpha} dv = \frac{2^{d-\alpha} \Gamma^{\frac{\alpha}{2}}}{(\alpha-2)c} \ {}_2F_1 \left[ 1,1-\frac{\alpha}{2}; \frac{\alpha}{2}; -\frac{d-\alpha}{c} \right] (A1.5.4)
\]

This concludes the proof. \( \square \)

**A1.6 Proof of Lemma 3.3**

When the shadowing variable \( \mu^{-1} \) for the desired signal is fixed, the desired signal power \( h \) follows an exponential distribution with mean \( \mu^{-1} \), denoted as \( h \sim \exp(\mu^{-1}) \) and the coverage probability at the distance \( r_1 \) to its serving BS becomes

\[
p_{c,1}(T|\mu^{-1},r_1) = E_{i,j=1,2} \left\{ \exp \left( -\mu r_1^{\alpha_i} \left( \frac{\sum_{j=1}^{L_i} l_j}{P_l} \right) T \right) \right\} \\
= \prod_{j=1}^{2} \left\{ E_{l} \left[ \exp \left( -\mu r_1^{\alpha_i} l_j T / P_l \right) \right] \right\} \\
= \prod_{j=1}^{2} \left\{ L_{l} \left( \mu r_1^{\alpha_i} T / P_l \right) \right\} (A1.6.1)
\]

202
where $\mathcal{L}_{I_j}(\cdot)$ is the Laplace transform of the aggregate interference $I_j$ from the tier $j$.

Then substituting $\mathcal{L}_{I_j}(s)$ (3.30) and $d_j$ (A1.3.3) into (A1.6.1) gives

$$p_{c,l}(T|\mu^{-1}, r_l) = \Pi_{j=1}^2 \left\{ \mathcal{L}_{I_j} \left( \mu^{-1} T / P_l \right) \right\}$$

$$= \Pi_{j=1}^2 e^{-\sqrt{\pi} \mu T / P_l} \sum_{n=1}^N \omega_n \frac{2 - a_j}{(a_j - 2)} 2 F_1 \left[ 1, 1 - \frac{2 - a_j}{2 a_j}, -\mu T / P_l, d_j^{-a_j} \exp(\sqrt{\pi} \sigma_j a_n) \right]$$

$$= \Pi_{j=1}^2 e^{-\sqrt{\pi} \mu T / P_l} \sum_{n=1}^N \omega_n \frac{2 F_j}{2} \frac{2}{(a_j - 2)} \exp(\sqrt{\pi} \sigma_j a_n) 2 F_1 \left[ 1, 1 - \frac{2 - a_j}{2 a_j}, -\mu T / P_l, d_j^{-a_j} \exp(\sqrt{\pi} \sigma_j a_n) \right] \frac{2}{a_j}$$

which concludes the proof of Lemma 3.3.

\[ \square \]

A1.7 Proof of Proposition 3.4

When the shadowing variable $\mu$ for the desired signal is fixed, the coverage probability $p_c(T|\mu^{-1}, r_l)$ has been derived in Lemma 3.3. Thus, the tier $l$ coverage probability at $r_l$ in terms of shadowing $\mu^{-1}$ is

$$p_{c,l}(T|\mu^{-1}, r_l) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} p_{c,l}(T|\mu^{-1}) e^{\sqrt{\pi} \sigma_l y} \exp(-y^2) dy$$

(A1.7.1)

By using the Gauss-Hermite quadrature in numerical analysis [66],

$$p_{c,l}(T|\mu^{-1}) = \sum_{m=1}^M \omega_m \sqrt{\pi} p_{c,l}(T|\mu^{-1}) = \sum_{m=1}^M \omega_m \sqrt{\pi} a_m, r_l) + O_M$$

(A1.7.2)

Substituting (3.31) into (A1.7.2) gives the results (3.33).

\[ \square \]
A2 Appendices to Chapter 4

A2.1 Proof of Lemma 4.1

Based on the biased association policy (3.3), the considered user at distance $r_1$ from the closest macro BS will associate to the first tier $l = 1$ if the distance from the closest small BS $r_2$ is greater than

$$R_2 \triangleq \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{a_2}} r_1^{\frac{a_1}{a_2}} \tag{A2.1.1}$$

Equation (A2.1.1) means that if there exists a small BS inside the circular area around the user with the radius $R_2$, the user will associate to the tier $l = 2$. This area is called the 2nd tier association circle, as shown in Fig. 4.1. Note that from (A2.1.1) $R_2$ is linear with $r_1$ when $\alpha_1 = \alpha_2$. The probability of a user at distance $r_1$ associating to the first tier is the probability $Pr\{X = 0\}$ where $X$ is the number of small BSs inside the circle of radius $R_2$ around the user, a Poisson random variable with parameter $\lambda_2^{BS} \pi \left[ \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{1}{a_2}} r_1^{\frac{a_1}{a_2}} \right]^2$.

$$Pr\{l = 1 | r_1\} = Pr\{r_2 > R_2\} = e^{-\lambda_2^{BS} \pi \left( \frac{B_2 P_2}{B_1 P_1} \right)^{\frac{2}{a_2}} r_1^{\frac{2a_1}{a_2}}} \tag{A2.1.2}$$

Since users in the selected hexagonal area are uniformly distributed, the approximate probability density function of the distance $r_1$ to the nearest macro BS $[60] [130]$ is

$$f_{r_1}(r_1) = \frac{2r_1}{R_1^2}, 0 < r_1 \leq R_1 \tag{A2.1.3}$$

where the hexagonal cell shape is approximated by a circle with the same area and the radius of the circle $R_1$ makes $\pi R_1^2 = 2\sqrt{3}R_M^2$.

$$R_1 = \sqrt{\frac{2\sqrt{3}}{\pi}} R_M \tag{A2.1.4}$$
Then the pdf of the distance $r_1$ in the tier $l = 1$ is

$$f_{r_1}(r_1, l = 1) = \frac{2r_1}{R_1^2} e^{-\lambda_2 BS \frac{P_2}{B_1 P_1}} \frac{2}{r_1^{2a_1}}$$  \hspace{1cm} (A2.1.5)

The pdf of the distance $r_1$ in the tier $l = 2$ is

$$f_{r_1}(r_1, l = 2) = \frac{2r_1}{R_1^2} \left( 1 - e^{-\lambda_2 BS \frac{P_2}{B_1 P_1}} \right)^2_{r_1^{2a_2}}$$  \hspace{1cm} (A2.1.6)

Integrating (A2.1.5) and (A2.1.6) over $r_1$ distribution in (A2.1.3) concludes the proof. \hfill \Box

**A2.2 Proof of Proposition 4.1**

The proof starts from two known analytical results. First, for a user associating to tier $l$ at $r_l$ from its serving BS and the desired signal is Rayleigh faded, it is shown in [20] that the coverage probability is

$$p_{c_l}(T | r_1) = \prod_{j=1}^{l-2} L_{i_j}(Tr_1^a p_i^{-1})$$  \hspace{1cm} (A2.2.1)

where $L_j(\cdot)$ denotes the Laplace transform of $I_j$, the interference from tier $j$.

Second, for the user associating to tier $l$, if the aggregate interference $I_j$ from tier $j$ is generated from a PPP with the intensity of $\lambda_j$ and the constraint of the minimal distance $d_{ij}$ to the considered user, the Laplace transform of $I_j$ is shown in (3.16),

$$L_{i_j}(s) = e^{- \frac{2\pi \lambda_j d_{ij}^{2-a} js p_j}{a-2}} 2F_1 \left[ \frac{1, -2, -2, a_j, -a_j, 0}{a_j, -s, p_j d_{ij}} \right]$$  \hspace{1cm} (A2.2.2)

Now consider the situation that the user at $r_1$ associates to the first tier, $l = 1$. In this situation, $I_1$ comes from all the other macro BSs excluding the closest one, which is approximated as the interference generated from a PPP of intensity $\lambda_1 = 1/(2\sqrt{3} R_1^2)$ outside the interference-free disc with the radius $d_{11} = 0.77 \times (2R - r_1)$ [79]. Thus $L_{i_1}(s)$ is formulated as (A2.2.1) where $j = 1$. 205
For the interference from the second tier, $I_2$, the closest small BS must be limited to $r_2 > R_2$, as discussed in (A2.1.1), due to the biased association policy. Thus $\mathcal{L}_{I_2}(s)$ is formulated by (A2.2.2) where $j = 2$ and $d_{12} = R_2$. Instituting the results of $\mathcal{L}_{I_1}(s)$ and $\mathcal{L}_{I_2}(s)$ into (A2.2.1) concludes the first part of Proposition 4.1.

Move on to the other situation where a user at $r_1$ associates to the second tier, $l = 2$. The user distance $r_2$ to its closest small BS must be less than $R_2$. This constraint is independent of the locations of other small BSs. Thus the pdf of $r_2$, denoted as $f_{r_2}(r_2)$, is a truncated Weibull distribution with the shape parameter of 2, as expressed in (4.10), based on the compound probability rule of independent events.

Now conditioning on the distance $(r_1, r_2)$, the coverage probability is

$$p_{c,l=2}(T|r_1, r_2) = \prod_{j=1}^{2} L_{I_j}(T r_1^{a_1} p_1^{-1})$$

(A2.2.3)

where the aggregate interference power from each tier $I_j$ is measured for the user at $(r_1, r_2)$. Here $I_1$ is different from that of the first situation (i.e. user at $r_1$ associates to the first tier), including the component of the interference from the closest macro BS, denoted by $I_{1,0}$, in addition to that from all the other macro BSs, $I_{1,1}$. That is, $I_1 = I_{1,0} + I_{1,1}$ and $L_{I_1} = L_{I_{1,0}} \times L_{I_{1,1}}$. With the assumption of Rayleigh fading and further considering the frequency reuse $\rho_1$ and transmission probability $\eta_1$,

$$L_{I_{1,0}} = \left(1 - \frac{\eta_1}{\rho_1}\right) e^0 + \frac{\eta_1}{\rho_1} \frac{1}{1 + s P_1 r_1^{-a_1}} = 1 - \frac{\eta_1}{\rho_1 + \rho_1 r_1^{-a_1}} s^{-1}$$

(A2.2.4)

And $I_{1,1}$ here is the same as $I_1$ in the first situation (i.e. user at $r_1$ associates to the first tier), which gives $L_{I_{1,1}}(s)$ as (A2.1.6) with $i = 1$ and $d_{21} = d_{11}$. Here $I_2$ is generated by a PPP with the interference-free disc of radius $r_2$, which gives $L_{I_2}(s)$ in (A2.2.2) with $i = 2$ and $d_{22} = r_2$.  

206
Substituting $\mathcal{L}_{I_1}(s)$ and $\mathcal{L}_{I_2}(s)$ into (A2.2.3) derives the coverage probability $p_{c,t=2}(T|r_1, r_2)$ for users at $(r_1, r_2)$.

Finally, integrating $p_{c,t=2}(T|r_1, r_2)$ in (A2.2.3) over the pdf of $r_2$ in (4.10), gives the coverage probability of the second tier for a user at $r_1$, as shown in (4.8), which concludes the second part of Proposition 4.1.

\[ \square \]

A2.3 Proof of Proposition 4.2

As stated in Proposition 3.1, the power distribution of the Rician faded desired signal, $f_X(x)$, can be approximated as an exponential series with $N$ terms, $g(x)$.

\[
  f_X(x) \approx g(x) = \sum_{n=1}^{N} w_n u_n e^{-u_n x}, x \in [0, V] \tag{A2.3.1}
\]

Thus, the coverage probability for a user associating to tier $l$ at $r_l$ from its serving BS and the desired signal is Rician faded is derived as follows, with the similar derivation as shown in Proposition 3.2,

\[
  p_{c,l=1}(T|r_1) = \sum_{n=1}^{N} w_n \left\{ \prod_{j=1}^{l-1} \mathcal{L}_{I_j} \left( u_n T r_l^{-\alpha_l} P_l^{-1} \right) \right\} \tag{A2.3.2}
\]

where $w_n$ and $u_n$ are derived by (3.10) for Rician factor $K_l$. $\mathcal{L}_{I_j}(\cdot)$ denotes the Laplace transform of $I_j$, the interference from tier $j$, which is formulated in Proposition 4.1. Thus using (A2.3.2) instead of (A2.2.1) and (A2.2.3), and following the other steps in Proposition 4.1, gives Proposition 4.2. \[ \square \]
A3 Appendices to Chapter 6

A3.1 Proof of Lemma 6.1

First the Laplace transform of $X^{BF}$ is derived as follows,

\[
\mathcal{L}_{X^{BF}}(s) = E_{X^{BF}}\left\{ e^{-sX^{BF}} \right\}
\]

\[
\overset{(a)}{=} E_{X^{BF}} \left\{ e^{-sX^{1-\cos(M_j\psi)}M_j(1-\cos\psi)} \right\}
\]

\[
\overset{(b)}{=} E_{\psi} \left\{ E_X \left[ e^{-sX^{1-\cos(M_j\psi)}M_j(1-\cos\psi)} \right] \right\}
\]

\[
\overset{(c)}{=} E_{\psi} \left\{ \left[ 1 + s \frac{1-\cos(M_j\psi)}{M_j(1-\cos\psi)} \right]^{-1} \right\}
\]

\[
= \int_{\psi} \left[ 1 + s \frac{1-\cos(M_j\psi)}{M_j(1-\cos\psi)} \right]^{-1} f_{\psi}(\psi) d\psi
\]  

(A3.1.1)

where (6.2) and (6.4) are applied in (a), (b) holds because because $X^{BF}$ is a function of $\psi$ and $X$ which are independent, and $X$ is exponential distributed due to Rayleigh fading which gives

\[
\mathcal{L}_{X}(s) = \left[ 1 + s \frac{1-\cos(M_j\psi)}{M_j(1-\cos\psi)} \right]^{-1}
\]  in (c).

The Laplace transform of the tier interference with BF is derived, by following the similar steps from (A1.2.1) to (A1.2.2),

\[
\mathcal{L}_{I_j}(s) = \exp \left\{ -2\pi \lambda_j \int_{d_j}^\infty \left[ 1 - \mathcal{L}_{X^{BF}} \left( sP_{j}r_j^{-\alpha_j} \right) \right] r_j d\psi \right\}
\]  

(A3.1.2)

Finally substituting (A3.1.1) into (A3.1.2) gives (6.5),

\[
\mathcal{L}_{I_j}(s) = \exp \left\{ -2\pi \lambda_j \int_{d_j}^\infty \left[ 1 + \int_{\psi} \left[ 1 + sP_{j}r_j^{-\alpha_j} \frac{1-0.5\cos(M_j\psi)}{M_j(1-0.5\cos\psi)} \right]^{-1} f_{\psi}(\psi) d\psi \right] r_j d\psi \right\}
\]
\[(d)\]

\[
\exp \left\{ -\pi \lambda_j \int_{d_j}^{\infty} \int_{\psi} \left( 1 - \left[ 1 + S P_j y \frac{\alpha_j}{2 M_j (1 - \cos(\psi))} \right]^{-1} \right) f_{\psi}(\psi) d\psi dy \right\}
\]

\[
= \exp \left\{ -\pi \lambda_j \int_{d_j}^{\infty} \int_{\psi} \left( \frac{\alpha_j}{2 M_j (1 - \cos(\psi))} \right) f_{\psi}(\psi) d\psi dy \right\}
\]

\[
= \exp \left\{ -\pi \lambda_j \int_{d_j}^{\infty} \int_{\psi} \left( 1 + \frac{\alpha_j y}{S P_j (1 - \cos(M_j \psi))} \right) f_{\psi}(\psi) d\psi dy \right\}
\]

\[\text{where } y = r_j^2 \text{ and } \int_{\psi} f_{\psi}(\psi) d\psi = 1 \text{ are used in (d) above.} \]

\[\square\]

**A3.2 Proof of Proposition 6.1**

For a user associating to tier \( l \) at \( r_l \) from its serving BS with BF such that the desired signals from multiple antennas experience fully correlated Rayleigh fading,

\[ h = \tilde{h} \cdot X \cdot BF_p(\psi = 0) = P_l r_l^{-\alpha_l} \cdot X \cdot M_l \]  

(A3.2.1)

where \( X \) is exponentially distributed with \( E(X) = 1 \) due to Rayleigh fading. \( BF_p(\psi = 0) = M_l \) in (A3.2.1) since the BF of the serving BS is pointed to the considered user, leading to \( \theta_1 = \theta \) and \( \psi = sin \theta_1 - sin \theta = 0 \).

Thus the coverage probability for the considered user at the distance \( r_l \) from the serving BS in the \( l \)th tier follows by definition,

\[ p_{c,l}(T|r_l) = E_{I_1, I_2} \left\{ P_T \left[ X > r_l^{\alpha_l} \left( \frac{\Sigma_{j=1}^{L} L_j}{P_l M_l} \right) T|r_l, I_1, I_2 \right] \right\} \]

\[= E_{I_1, I_2} \left\{ \exp \left( -r_l^{\alpha_l} T(M_l P_l)^{-1} \Sigma_{j=1}^{L,2} L_j \right) \right\} \]

\[= \prod_{j=1}^{\infty} L_j \left( T_{j} \right) \]  

(A3.2.2)
where in (e), $X$ is the fading power gain of the desired signal. (f) holds since $X$ is exponentially distributed due to Rayleigh fading. $\mathcal{L}_{I_j}(s) \triangleq E\{e^{-sI_j}\}$ denotes the Laplace transform of $I_j$ in (g) that holds due to the independence of $I_1$ and $I_2$.

Now consider the situation that the user at $r_1$ associates to the first tier, $l = 1$. In this situation, $I_1$ comes from all the other macro BSs excluding the closest one, which is seen as the interference generated from a PPP of intensity $\lambda_1$ outside the interference-free disc with the radius $d_{11}$. Thus $\mathcal{L}_{I_1}(s)$ is formulated by (6.5) where $j = 1$. For the interference from the second tier, $I_2$, the closest small BS must be limited to $r_2 > R_2$, as discussed in (A2.1.1), due to the biased association policy (3.3). Thus $\mathcal{L}_{I_2}(s)$ is formulated by (6.5) where $j = 2$ and $d_{12} = R_2$. Instituting the results of $\mathcal{L}_{I_1}(s)$ and $\mathcal{L}_{I_2}(s)$ into (A3.2.2) concludes the first part of Proposition 6.1.

Move on to the other situation where a user at $r_1$ associates to the second tier, $l = 2$. The user distance $r_2$ to its closest small BS must be less than $R_2$. This constraint is independent of the locations of the other small BSs. Thus the pdf of $r_2$, denoted as $f_{r_2}(r_2)$, is a truncated Weibull distribution with the shape parameter of 2, as expressed in (4.10), based on the compound probability rule of independent events. Now conditioning the considered user on the distance $(r_1, r_2)$, the coverage probability is

$$p_{c,l=2}(T|r_1, r_2) = \prod_{j=1}^{2} \mathcal{L}_{I_j} \left( \frac{T r_{1,j}^{a_j}}{M t_{p_j}} \right)$$

(A3.2.3)

Note that different from $I_1$, the first tier interference for users associating to the first tier, the interference for users associating to the second tier comprises the interference component from the closest macro BS (denoted by $I_{1,0}$) and from all the other macro BSs (denoted by $I_{1,1}$). That is, $I_1 =$
$I_{1,0} + I_{1,1}$ thus $\mathcal{L}_{I_1} = \mathcal{L}_{I_{1,0}} \times \mathcal{L}_{I_{1,1}}$. With the assumption of the fully correlated Rayleigh fading among multiple antennas, the Laplace transform of $I_{1,0}$ is

$$
\mathcal{L}_{I_{1,0}}(s) = \left(1 - \frac{\eta_1}{\rho_1} \right)e^0 + \frac{\eta_1}{\rho_1} \times E_{X_{BF}} \left\{ e^{-sP_1r_1^{-\alpha_1}}X_{BF} \right\}
$$

$$(i) = 1 - \frac{\eta_1}{\rho_1} + \mathcal{L}_{X_{BF}}(sP_1r_1^{-\alpha_1})
$$

$$(j) = 1 - \frac{\eta_1}{\rho_1} \int_\psi \left\{ 1 - \left[ 1 + sP_1r_1^{-\alpha_1} \frac{1 - \cos(M_j \psi)}{M_j (1 - \cos \psi)} \right]^{-1} \right\} f_\psi(\psi) d\psi
$$

$$(k) = 1 - \frac{\eta_1}{\rho_1} \int_\psi \left\{ 1 + \frac{r_1^{\alpha_1}M_j (1 - \cos \psi)}{sP_1 (1 - \cos(M_j \psi))} \right\}^{-1} f_\psi(\psi) d\psi \quad (A3.2.4)
$$

where the impact of the frequency reuse $\rho_1$ and transmission probability $\eta_1$ on the Laplace transform of $I_{1,0}$ is formulated in (h), (i) is given by the definition of Laplace transform of $X_{BF}$, (A3.1.1) is used in (j) and simple algebraic manipulation gives (k).

Laplace transform of $I_{1,1}$, $\mathcal{L}_{I_{1,1}}(s)$, is given by (6.5) with $i = 1$ and $d_{21} = d_{11}$, the same as $I_1$ in the first situation (i.e. user at $r_1$ associates to the first tier). $I_2$ is generated by a PPP with the interference-free disc of radius $r_2$, whose Laplace transform $\mathcal{L}_{I_2}(s)$ is given by (6.5) with $i = 2$ and $d_{22} = r_2$. Substituting $\mathcal{L}_{I_1}(s)$ and $\mathcal{L}_{I_2}(s)$ into (A3.2.3) derives the coverage probability $p_{c,i=2}(T|r_1,r_2)$ for the considered user at $(r_1, r_2)$.

Finally, integrating $p_{c,i=2}(T|r_1,r_2)$ over the pdf of $r_2$ in (4.10), gives the coverage probability of the second tier for a user at $r_1$, shown in (6.7), this concludes the second part of Proposition 6.1. □

### A3.3 Proof of Lemma 6.2

The received power $X_{TD}$ from all co-located antennas at a BS is Gamma distributed, $X_{MRT} \sim \gamma(M_j, 1/M_j)$, whose Laplace transform is
\( \mathcal{L}_{X^\text{MRT}}(s) = \left(1 + \frac{s}{M_j}\right)^{-M_j} \) \hspace{1cm} (A3.3.1)

Following the similar derivation of (A1.2.2), the Laplace transform of the aggregate interference is

\[
\mathcal{L}_i(s) = \exp\left\{-2\pi \lambda_j \int_{d_j}^{\infty} \left[ 1 - \mathcal{L}_{X^\text{MRT}}(s^P r_j^{-\alpha_j}) \right] r_j \, dr \right\}
\]

\[
= \exp\left\{-2\pi \lambda_j \int_{d_j}^{\infty} \left[ 1 - \left(1 + \frac{s^P r_j^{-\alpha_j}}{M_j}\right)^{-M_j} \right] r_j \, dr \right\}
\]

\[
= \exp\left\{-\pi \lambda_j \int_{d_j}^{\infty} \left[ 1 - \left(1 + \frac{s^P y^{-\alpha_j}}{M_j}\right)^{-M_j} \right] dy \right\} \hspace{1cm} (A3.3.2)
\]

where \( y = r_j^2 \) is used in \((l)\). This concludes the proof. \( \square \)

A3.4 Proof of Proposition 6.2

For the desired signal power from all the antennas at the service BS in tier \( l \), \( X^{TD} \) follows the Gamma distribution \( X^{TD} \sim \gamma(M_j, 1/M_l) \), where the shape parameter is \( M_l \) and the scale parameter \( \theta_r = 1/M_l \). Correspondingly, the coverage probability for the considered user at the distance \( r_l \) from the serving BS in the \( l^{th} \) tier is

\[
p_{c,l}(T|r_l) = E_{t_1,t_2} \left\{ P_r \left[ X^{TD} > r_l^{M_l} \left( \frac{\sum_{j=1}^{L_l} l_j}{p_t} \right) \right] T|r_l, l_1, l_2 \right\}
\]

\[
= \left( \begin{array}{c}
E_{t_1,t_2} \left\{ \sum_{i=0}^{M_l-1} \frac{s^l}{i!} \left( \frac{r_l}{p_t} \sum_{j=1}^{L_l} l_j \right)^{i} e^{-\sum_{j=1}^{L_l} l_j} \right\} \\
E_{t_1,t_2} \left\{ \sum_{i=0}^{M_l-1} \frac{s^l}{i!} \left( \sum_{j=1}^{L_l} l_j \right)^{i} e^{-\sum_{j=1}^{L_l} l_j} \right\}
\end{array} \right) \hspace{1cm} (A3.4.1)
\]
where \( F_\gamma(x; M, \theta) = 1 - \sum_{i=0}^{M-1} \frac{1}{i!} (\frac{x}{\theta})^i e^{-\frac{x}{\theta}} \) is applied in (m) since \( X^{TD} \) follows the Gamma CDF with a positive integer shape parameter \( M, s = \frac{Ml_1^{a_l}}{p_l} \) is set in (n), the order change of summation and expectation gives (o), (p) holds due to the Laplace transform property (differentiation in the s-domain): \( E_{ij}\{I^j e^{-sl}\} = \frac{d^i}{ds^i} L_i(s) \) where \( I = \sum_{j=1}^l I_j \).

Now the situation that the user at \( r_1 \) associates to the first tier, \( l = 1 \), and the other situation that the user at \( r_1 \) associates to the second tier, \( l = 2 \), are considered respectively, by following the similar derivation steps in the proof of Proposition 6.1. Briefly, in the first situation, \( \mathcal{L}_{I_1}(s) \) and \( \mathcal{L}_{I_2}(s) \) are both formulated by (6.10) where \( j = 1 \) and \( d_1 = d_{11} \) for \( \mathcal{L}_{I_1}(s) \) and \( j = 2 \) and \( d_2 = d_{12} \) for \( \mathcal{L}_{I_2}(s) \).

Instituting the results of \( \mathcal{L}_{I_1}(s) \) and \( \mathcal{L}_{I_2}(s) \) into (A3.4.1) concludes the first part of Proposition 6.2. In the second situation where a user at \( r_1 \) associates to the second tier, \( l = 2 \), the interference from the first tier \( I_1 \) comprises the interference component from the closest macro BS (denoted by \( I_{1,0} \)) and from the other macro BSs (denoted by \( I_{1,1} \)). With the assumption of the independent Rayleigh fading among multiple antennas with TD applied, the Laplace transform of \( I_{1,0} \) is

\[
\mathcal{L}_{I_{1,0}}(s) = \left( 1 - \frac{\eta_1}{\rho_1} \right) e^0 + \frac{\eta_1}{\rho_1} \times E_{X^{TD}} \left\{ e^{-sP_1 r_1^{-a_1} X^{TD}} \right\}
\]

\[
= 1 - \frac{\eta_1}{\rho_1} \times \left[ 1 - \mathcal{L}_{X^{TD}}(s P_1 r_1^{-a_1}) \right]
\]

\[
= \left( 1 - \frac{\eta_1}{\rho_1} + \frac{\eta_1}{\rho_1} \left( 1 + \frac{s P_1 r_1^{-a_1}}{M_j} \right)^{-M_j} \right)
\]

(A3.4.2)

where the impact of the frequency reuse \( \rho_1 \) and transmission probability \( \eta_1 \) on Laplace transform of \( I_{1,0} \) is formulated in (q), the definition of Laplace transform of \( X^{TD} \) is used in (r), and (A3.3.1) is used in (s). Further, \( \mathcal{L}_{I_{1,1}}(s) \) is given by (6.10) with \( j = 1 \) and \( d_j = d_{21} \) thus \( \mathcal{L}_{I_1}(s) \) is achieved by
\( \mathcal{L}_{l_1} = \mathcal{L}_{l_{1,0}} \times \mathcal{L}_{l_{1,1}} \). Further conditioning on \( r_2 \), \( \mathcal{L}_{l_2}(s) \) is given by (6.10) with \( j = 2 \) and \( d_j = d_{22} \), thus substituting the derived \( \mathcal{L}_{l_1}(s) \) and \( \mathcal{L}_{l_2}(s) \) into (A3.4.1) gives the coverage probability \( p_{c,l=2}(T|r_1, r_2) \) for the considered user at \( (r_1, r_2) \). Finally, integrating \( p_{c,l=2}(T|r_1, r_2) \) over the pdf of \( r_2 \) in (4.10), gives the coverage probability of the second tier for a user at \( r_1 \), shown in (6.13), that concludes the second part of Proposition 6.2. \( \square \)